Volume V. *August, 1897.* *Number 2.*

THE

PHYSICAL REVIEW.

ON THE CONVERSION OF ELECTRIC ENERGY IN DIELECTRICS.[1] III.

(*Conclusion.*)

BY RICHARD THRELFALL.

IN a particular test the primary potential difference was 105 electrostatic units, and the potential difference across the armatures, as tested by sparking distance, only 10 electrostatic units, instead of 37, as it should be according to this theory. The difference is to be explained by losses from brush discharges and imperfections in the theory.

If we consider the electric force at the center of the armatures as made up of components X and Y at right angles to each other, we may write

$$X = \frac{V_2 - V_1}{d},$$

$$Y = \frac{V_2 - V_3}{d},$$

where d is the distance between opposite armatures. We may write, then,

$$X = Ae^{-\lambda t} \sin(\mu t - \phi),$$

$$Y = Ae^{-\lambda t} \cos(\mu t - \phi),$$

where

$$A = \frac{E_0}{\sqrt{2} \cdot d}.$$

The field between the armatures is therefore a rotating one, and the value of the electric force at the center can be represented by the radius vector of a spiral, the magnitude ρ of the radius vector at any time being given by

$$\rho = Ae^{-\lambda t}.$$

[1] Continued from the PHYSICAL REVIEW, Vol. V., p. 46.

Now $$\int^{\infty} p\,dt = \frac{A}{\lambda},$$

and if T is the interval between two sparks across the knobs in the primary circuit, the mean value of the force will be $\frac{A}{\lambda T}$.

Putting in the value of A, we get

$$\text{mean force} = \frac{E_0}{\sqrt{2}\,d\lambda T}.$$

With respect to the above investigation, it is to be noted that it is tacitly assumed (1) that the currents are distributed uniformly throughout the conductors, and (2) that the conductors are short compared with the wave length. An idea of the error introduced by the former supposition is to be obtained from the previous footnote: it practically gives rise to no appreciable difference. With regard to the latter, all the conductors are short compared with the wave length, except the coils of the transformer of the Ebert apparatus. This does not give rise to any appreciable error, however, for I found the secondary potential difference almost exactly what it ought to have been.

With regard to the effect discovered by Jaumann, relating to the influence of a small impressed oscillatory potential difference at the spark gap, we may note that in the present case, either in the primary or in the secondary, the potential difference rises rapidly, but still gradually, to its maximum value, — a state of affairs which does not correspond with that treated experimentally by Jaumann. Other observers, however, have indicated that oscillatory discharges of very high frequency take place more easily than slower ones; but I am not able to discover how far the observations refer to the first spark as distinguished from those following it at very small intervals of time. Suppose, however, that such an effect really exists and operates on the first maximum in the armature (phasing) terminals. The result would be still further to increase the difference between the observed and calculated potential differences. Now the observed potential difference, as deduced from "steady spark length," was about one-quarter of the calculated potential difference. It will be presently shown that no reasonable reduction of the value thus obtained could possibly account for the absence of any observable rotation in the cases examined, assuming that the losses are the same at these high frequencies as they are at lower speeds. As an illustration of the method of making an experiment in the rapid field, I will select for description some observations on an ebonite cylinder, made from the same kind of ebonite as was examined in the slower fields. A fine thread having been selected, the ebonite cylinder was mounted and dried for several days by phosphorus pentoxide. The Ebert apparatus was fed by sparks from the large induction coil at the rate

of an interval of 0.0014″ between each spark. The sparks were examined by a revolving mirror, and the resistance of the primary of the induction coil was so adjusted as to allow the sparks to pass freely. It resulted from a previous experiment that the sparks only occurred on the make and not on the break of the primary circuit. The spheres of the Ebert apparatus were 3 cm. diameter of heavily gilt and polished brass. The rotating mirror (hand turned) disclosed that each spark was multiple. Since the period of oscillation was too rapid for the separate oscillations to be separated by the mirror at the speed at which it was driven, it was inferred that the enormous resistance, self-inductance, and capacity of the secondary of the induction coil was giving rise to discontinuous discharge in the well-known manner. The period quoted is merely that of the spark groups, or breaks of the induction coil circuit; consequently it appears reasonable to infer that the number of effective sparks was about four times as great as the number resulting from the period quoted. We are, however, in search of a minimum possible value for the intensity in the rotating field, and consequently the sparks will be reckoned at one only per break of primary. The spark length between the balls of the Ebert apparatus was 1 cm., corresponding to 105 electrostatic units potential difference.

The spark length between 1 cm. balls attached to the secondary condenser was such as to indicate a potential difference exactly half of this, within the limits of errors of observation. The potential differences were deduced by a reference to Baille's results (J. J. Thomson, *Recent Researches in Electricity and Magnetism*, p. 77).

The phasing condenser plates were 15 cm. diameter, and when the phasing potential differences were equal they were 2.3 cm. apart. When the rotating field cell was connected with the wires and keys, readjustment was obtained by keeping the resistance constant and increasing the plate distance to 3.1 cm. This shows that the capacity of the cell was by no means to be neglected in reality, though we are more or less forced to neglect it in our calculations. This disturbance constitutes the most serious objection to the interpretation of the results, but it is not, after all, very large. The potential difference of the phasing terminals, with cell attached, was tested by observing the spark length between spheres 0.4 cm. diameter. One of these spheres was mounted on the slide of a measuring microscope (with, of course, the necessary insulation), and was found to yield a spark length of 0.06 cm. This corresponds to about 10 electrostatic units of potential difference. When the cell was disconnected, the potential difference rose so as to give a spark length of 0.14 cm., the difference being attributable to the inevitable loss by brush discharge, which it is practically impossible to avoid with the complexity of wires and keys actually employed.

Calculating the mean force by help of the formula arrived at in the mathematical discussion, we have

$$F = \frac{E_0}{\sqrt{2}\, d\lambda t}.$$

The data, besides those already given, used in the calculation of λ, are:

$E_0 = 105$ (electrostatic). $\quad \lambda = 366.$

$t = 14 \times 10^{-4}$ seconds. $\quad d = 1.3$ cm.

The resulting value of F is about 56 electrostatic units. Applying an "internal screening factor," this reduces to about 16.8 electrostatic units actually active in the mass of the cylinder of ebonite.

But we have seen that the want of uniformity in the distribution of the currents in the conductors may increase λ rather less than three times. Hence, taking 3λ instead of λ, the external force becomes 18, say, and the internal force, say, 5.

But the observed external force is $\frac{10}{1.3} = 7.7$, say, instead of 18, and the resulting external force is 2.3, say.

Using these latter numbers, we have the energy loss per unit volume by the previous investigation in slow fields

$$W = 0.03\, F^{1.85},$$

using the formula applicable to the smaller sphere. In this case the volume of the ebonite cylinder was 0.36 cc., so that the energy expended and lost becomes 0.4 erg per revolution. This implies a possible rise of temperature sufficient to melt paraffin in, say, about ten minutes, but the ebonite was cemented on by paraffin, and the field was often in action for ten minutes and more, but the ebonite never fell off. It is quite a question whether a temperature method might not yield instructive results if used to check the dynamometric method. Of course, as the apparatus was arranged, no temperature tests were possible. In order to compute what angular displacement of the cylinder is to be expected, the following data are necessary.

Time of double vibration of cylinder of moment of inertia, 0.08 (g.cm.^{2}); with thread employed $= 43''$, hence coefficient of torsion of thread is $1.7 \times 10^{-3} = \tau$, and if ϕ is the angular rotation of the cylinder in radians, we have $2\pi\tau\phi = W$.

This gives, with above minimum value for W,

$$\phi = 37 \text{ radians.}$$

Now, as the scale was set up at a distance of 150 cm., a 5 cm. deflection, which I shall show directly could not have been overlooked, would correspond to a rotation of the ebonite of 0.016 radian.

I will now show that though low frequency results would lead to an angular motion of the light spot of 74 radians, the observed angular motion was either absent or less than 0.032 radian; that is, $\frac{1}{2312}$ of the deflection which would have been observed had the low frequency loss persisted.

There was some fear that the water resistance might introduce a sensible capacity at the terminals, and consequently, in this case as in all others, three experiments were made.

(1) With the resistance and capacity in the phasing arrangement as described.

(2) With the resistance augmented by a small impedance, viz. the coil of a low resistance galvanometer.

(3) With two dead resistances of water — one the resistance employed in the other arrangements, and the second adjusted to equality by adjusting the spark lengths.

The results of many sets of measurements are collected for reference in Table A.

It may reasonably be inferred from the irregularity of the results that the deflections, such as they were, were the result of unequal electrifications of the ebonite, and this is supported by the fact that the deflections were not generally produced with the suddenness which should have characterized the real effect when the coil was suddenly started. On several occasions, also, observations were interrupted by the cylinder becoming electrified permanently to such an extent as to throw the reading right off the scale, where it remained long after the coil was stopped.

It may, perhaps, be urged that owing to the capacity of the water resistance, or of the leads, or of the armatures, a phase difference of 90° was not in reality set up. This is, of course, possible, but it is hardly likely that in the three cases examined the phase difference would have been 0° or 180° in each case, and yet this is the position we must take up if we assume that no deflection was observed, because in fact the field did not rotate. In fine, the margin is so great that we may admit that the vibration was nearly plane polarized in all three cases without affecting the conclusion.

Similar experiments were performed on flint glass and on sulphur, containing a great deal of the insoluble variety, and therefore suitable for the detection of any effect, but the results with both these substances were practically identical with the results as quoted.

These experiments were made before the unique properties of selenium had been discovered, and the apparatus was in use for other purposes, or I should, most certainly, have examined this curious substance.

We may now summarize the results of this investigation in Tables B and C.

TABLE A.

EBONITE CYLINDER IN HIGH FREQUENCY FIELD.

Clockwise rotations marked +; counter-clockwise marked −.

Position of the two six-point keys.	Position of the rotation reversing key.	Expected direction of rotation. Clockwise marked +.	1	2	3	4	5	6	7	8	9	10	11	12
			Deflections in centimeters. Phasing, condenser, and resistance adjusted.	Deflections in centimeters. Two equal resistances in phasing apparatus.	Deflections in centimeters. Phasing as in column 1. Normal	Deflections in centimeters. Inductance added to resistance and condenser.	Deflections in centimeters. Resistance, induction, and condenser. as in 4.	Deflections in centimeters. Two resistances.	Deflections in centimeters. Resistance and capacity. Normal.	Deflections in centimeters. Resistance, inductance, and capacity.	Deflections in centimeters. Two resistances only.	Mean deflections in centimeters. Normal arrangement.	Mean deflections in centimeters. Resistance, inductance, and capacity.	Mean deflections in centimeters. Two resistances.
A	*V*	−	0.0	+ 2.5	+ 0.8	0.0	− 2.0	− 1.0	− 2.0	− 1.0	− 1.0	− 0.62	− 0.1	+ 0.2
A	*H*	+	− 13.2	− 13.5	− 11.6	− 18.0	− 25.0	− 20.0	− 17.0	− 16.0	− 15.0	− 14.0	− 20.0	− 16.0
B	*H*	−	+ 21.5	+ 21.0	+ 20.0	+ 35.0	+ 20.0	+ 14.0	+ 13.0	+ 12.0	+ 10.0	+ 18.1	+ 22.0	+ 15.0
B	*V*	+	0.0	− 0.6	+ 0.3	− 1.5	− 1.5	− 2.0	− 1.0	− 1.2	− 1.4	− 0.2	− 1.4	− 1.3

TABLE B.

Description.	Energy loss and electric intensity relation.	Percentage loss $F=1$.	Percentage loss formula.	Frequency.	Effect of change of frequencies.
Ebonite "Long *A*"	$W = 0.029\ F^{1.765}$	21.2	$21.2\ F^{-0.235}$	0.15	Very slight increase with speed.
Silvertown best.					
Large sphere	$W = 0.017\ F^{1.85}$	—	—	—	Nil.
Smaller sphere	$W = 0.03\ F^{1.85}$	—	—	—	Nil.
Flint glass spheroid	$W = 0.038\ F^{1.92}$	14	$14\ F^{-0.08}$	18.8	Nil.
Sulphur spheroid:					
A, aged monoclinic; not especially dried	$W = 0.0139\ F^{1.91}$	11.05	$11.05\ F^{-0.09}$	22	Large decrease.
C, dried	Index $n = 1.5$	Too small for measurement			
K, 0.7 per cent insol.: perfectly dry	$W = 0.00048\ F^{1.857}$	0.0038	$0.0038\ F^{-0.143}$	18	Small increase.
Paraffin M.P. 50°:					
Spheroid *A*	Too small to measure by thread used for ebonite				
B, M.P. 52°	Very small loss indeed Index 1.95				
C	$W = 0.00138\ F^{1.56}$	1.51	$1.51\ F^{-0.44}$	12.5	Considerable decrease.
C	$W = 0.0008\ F^{1.56}$	0.88	$0.88\ F^{-0.44}$	27.3	Considerable decrease.
Various spheres	Index 1.5 to 1.6		Amplitude not reduced.		

TABLE C.

Description.	Energy loss and electric intensity relation.	Per cent formula.	Change with speed variation.
Selenium:			
Commercial A	Immeasurably large loss.		
Commercial B	Index 1.96	Similar to next, but 3.4 times larger at 5000 volts.	Nil.
Commercial C	Index 1.96	Amplitude about same as next, *D*.	Small increase.
Pure D	$W = 4.6\ F^{1.85}$	More than 400 per cent.	No observations.
Resin: Many spheres.	Index varied from 2.8 to 1.86. Loss $F = 1$ from 4 to 18 per cent.		Small increase?
	Rapid field rotating between 10^6 and 10^7 per second. Ebonite, glass, and complex sulphur gave no loss at all, even though the electric intensity employed was considerably greater than in the other experiments here summarized.		
	Heterogeneous conductors. Paraffin and resin, mixed with large quantities of graphite, gave results not much greater than when the pure dielectrics were employed. The index in these cases was about 2.		
	Conductors: aluminium, paraffin coated with tin-foil, arc light carbon. Resin and paraffin, made conducting with graphite, gave no effect whatever in slow fields.		
	Bisulphide of carbon, benzene, and kerosene were not set in rotation by the rotating field.		

It results from the above investigation (1) that the effect of moisture is very far from being eliminated by the Arno method, though the circumstances are very favorable in the sense that moisture should have a smaller effect than in experiments involving rapid space variations of the electric forces.

Nevertheless, prolonged drying will bring the insulators examined down to a steady state, and a residual hysteresis effect makes its appearance.

This effect can be represented over a range of potential differences of from 0 to 6000 volts with an accuracy of the order of the accuracy of observation by an expression of the form $W = aF^n$. No physical conclusion, however, can be drawn from such a formula, for it merely sums up the results of many complex processes; and it is notorious that a formula of

the kind quoted can be made to fit almost any curve over a short range. The values of n vary from about 1.5 to about 1.95 for ordinary "homogeneous" dielectrics. When dielectrics are made heterogeneous by the addition of graphite, the index rises to 2 and over, and the effects become irregular. This result was not unforeseen, for I had previously found that sulphur conducts discontinuously, and conduction in heterogeneous dielectrics is probably an important feature of the phenomenon. The rise of the index above 2 is to be explained on Hess's theory by the remark that the conduction does not follow Ohm's law; and in terms of the experiments by the remark that if the rate of subsidence of the polarization of the dielectric due to conduction is comparable with the period of field rotation, the energy put into the dielectric during one rotation is quite indeterminate, and the drag depends on the angle between the intensity and the polarization, which in turn depends on the non-ohmic conductivity.

(2) The hysteresis so discovered is quite constant for any given specimen, but is very variable as we pass from specimen to specimen, the index of the empirical expression of the variation being much more constant than the coefficient. It is known that very small differences in physical condition produce immense variations in the hysteresis loss of iron.

The hysteresis of dielectrics resembles the magnetic hysteresis of iron in this respect, but the phenomenon is on the whole still more strongly marked.

(3) The great differences between the values of specific inductive capacity assigned to certain substances by different observers, as well as the results here obtained, suggest that perhaps it is nearly as futile to treat the specific inductive electric capacity of a "substance" as distinguished from a "specimen" as constant, as to extend the same treatment to the specific inductive magnetic capacity of iron, even under given magnetic forces.

(4) The mechanics of the problem of the rotating field require that there should be an angular interval between the direction of the electric polarization and the direction of the electric force, in order that there may be any drag on the suspended spheroid. The problem is essentially similar to the problem of the couple exerted by the moon on the earth about the axis of the latter, and due to the want of correspondence between the phases of the tide and of the tide-raising force.

The fact that there is a drag on the spheroid exerted by the rotating field implies either (1) that dielectric polarization takes an appreciable time to become established, or (2) that dielectric polarization does not die away in simple proportion as the force is reduced. The present experiments do not enable us to discriminate between the phenomena, and it is most probable indeed that one implies the other. The experiments of

observers on the variation of specific inductive capacity, expressed as a function of the period of charge, lead to the same result, but I believe that they have all been affected to some extent by surface conduction.

(4) When I commenced these experiments I hoped to increase the angle between the direction of polarization and the direction of electric force by increasing the rate of rotation of the electric field. Assuming for a moment that the time required for the establishment of the polarization is merely the time required for the inwards sideways motion of the Faraday tubes, we see that for experiments of the frequency adopted in the mechanically rotating cell, the polarization may be considered as established instantaneously. In this case, the drag is due to the persistence of the polarization, and would become a maximum when the rotation was so fast as to make the angle between the electric force and the polarization vectors exactly 90°. A reference to the summary will make it clear that when the dielectrics were dry the drag was either inappreciably affected by increasing the rate of rotation or was increased thereby. At a frequency about a million the effect vanished. There is clearly a speed of maximum effect somewhere between a frequency of rotation of 30 and of a million per second. If I had had a high frequency alternator at my disposal, I should certainly have attempted to run this down, for I think that the measurement of the period of persistence which would be deduced could not fail to give most important information.

(5) The Maxwell suggestion as to the cause of residual discharge does not, I think, account for the results of these experiments. My former experiments on sulphur (*loc. cit.*) show most unmistakably that the break-down of polarization is of an electro-chemical nature, and this fact is sufficient to account for residual charge if the ions are not free to reach the electrodes. The present series of experiments indicates how the time required for this break-down may be measured by any one fortunate enough to possess a couple of alternators of high and low frequency, respectively.

In conclusion, I desire again to thank those who have assisted me, — Mr. Alfred Pepper, in the earlier experimental work, Professor Lyle, who supplied the substance of the mathematical investigation, and Mr. Pollock, who enabled me to apply the same, and who kindly checked a typical set of observations and reductions.

UNIVERSITY OF SYDNEY, NEW SOUTH WALES,
Nov. 23, 1896.

A DETERMINATION OF THE SPECIFIC RESISTANCE AND TEMPERATURE COEFFICIENT OF OIL IN THIN FILMS, AND THE APPLICATION OF THESE RESULTS TO THE MEASUREMENT OF THE THICKNESS OF THE OIL FILM IN JOURNAL BOXES.

By F. L. O. Wadsworth.

Introductory Note.

THE results presented in the following paper were obtained several years ago while I was a student at the Ohio State University. The second part of the investigation had just been begun when the investigation was interrupted by my departure from the University. Since then I have always hoped to be able to carry the work to completion, but the continued pressure of other duties has so far prevented me from doing this. As I am now permanently engaged upon other entirely different lines of investigation, I have decided to publish the account of the work already completed in the hope that the results so far obtained may not be uninteresting in themselves.

The question of how to determine the thickness of oil in a journal box under actual conditions of work, and find how it varies with varying conditions of speed and pressure, is a most interesting and important one to the mechanical engineer; but it has not yet, so far as I am aware, been satisfactorily answered. The method proposed at the end of the paper seems a promising one, to judge from the few preliminary experiments which were made; and I hope some one may be sufficiently interested in the problem to continue the work along this line.

Considering the length of time that has now elapsed since the article describing the results of the measurement of the resistance of films was written, it has been thought best to leave it in the original

form except for a few minor changes in wording or arrangement. Footnotes have been added wherever it was thought desirable to make modifications or corrections.

YERKES OBSERVATORY, UNIVERSITY OF CHICAGO.
March, 1897.

Very little work seems to have been done in determining the specific resistance and temperature coefficient of the various kinds of oil. The only experiments which the writer has been able to find are those of Brooks[1] on the *relative* resistances of a sample of paraffine oil for various temperatures between 4.4 C. and 93.3 C.[2] The experiments were made on a sheet of oil 3 mm. thick and about 2.3 sq. cm. in cross section. No data, however, were given from which the absolute resistance could be calculated, nor was it stated what particular kind of paraffine oil was used.

In the experiments recorded below, which have been undertaken as preliminary to the determination of the thickness of oil in journal bearings, the oil used was the best quality of commercial sperm oil, in which zinc had been placed for over a year, in order to free it from any acid which it might contain. In addition to the determination of the absolute specific resistance and temperature coefficient, another object of these experiments was to determine whether there was any variation of specific resistance with variation in the thickness of the film,[3] or with the strength of current used.

Apparatus and Method.

In order to obtain a thin uniform film of oil of a determinate thickness, the following arrangement was used:

A pair of fine (Brown & Sharpe) surface plates, size about 4 by 6 inches, were placed one above the other with their surfaces separated a small distance by pieces of thin sheet rubber or parafined paper.[4] At

[1] Wiedmann's Electricitat, Vol. II., p. 5.

[2] Since these experiments were made a considerable amount of work has been done in determining the insulation resistance of various kinds of oil, particularly for the very high electromotive forces used in alternate current transmissions.

[3] I did not learn of Reinhold and Rucker's results on the electrical resistance of thin soap films until some time after this work had been completed. (See also subsequent foot note on p. 93.)

[4] A method very similar to this in principle has since been used by Prof. Threlfall for obtaining the resistance of a thin film of sulphur. Phil. Mag. Vol. 38, p. 452. 1889.

first the capillary attraction between the plates was relied upon to keep the film of oil uniformly spread between the two adjacent surfaces, but this being found sometimes unreliable, the two plates were immersed in a sheet iron tank filled with the oil under examination to a height just above the surface of the under plate (See Fig. 1). In order to avoid any danger of air-bubbles in the film of oil between the plates, the upper plate was removed before beginning each series of experiments, and the oil heated to a temperature of about 50°. The pieces of insulating material were then placed in position, and the upper plate let down upon the lower one, care being taken to make the edges of the two plates exactly coincide. To drive out the bubbles of air which might cling to the surface of the upper plate when it was let down, the two plates were brought together with a quick hinge-like motion. The plates and the tank of oil were then allowed to cool to the temperature of the room before beginning the measurements. The temperature was observed by means of two thermometers, one immersed in the oil of the tank, the other immersed in oil contained in the cavity of the upper surface plate. The resistance of the film of oil between the two plates was measured by comparing it with a 250,000 ohm box, by the method of deflections. The arrangement of the circuit is shown in Fig. 1, where

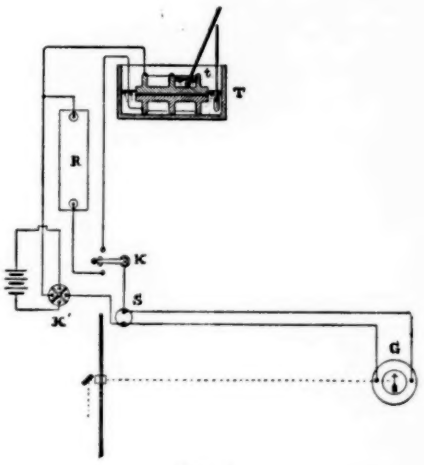


Fig. 1.

G is a Thomson galvanometer of 6,000 ohms resistance.

R is a 250,000 ohm box (of platanoid correct at 18° C., temp. coefficient +.0002.)

T is a tank containing the two surface plates, t_1 and t_2, arranged as before described.

K is a three way key for changing from the circuit through R to the circuit through t_1 and t_2.

K' is a reverse key for reversing the current in the battery circuit.

S is a shunt in the galvanometer circuit.

Let G be the resistance of the galvanometer; R the resistance of the box; Q the resistance of the oil film; B the resistance of the battery and rest of the circuit (which may be neglected in comparison with either Q or R); S_1 and S_2 the shunts used respectively with R and Q; d_1 the deflection of the galvanometer when R is in circuit and d_2 the deflection when Q is in circuit. Then if d_1 and d_2 are not greatly different, it has been proven

$$\frac{R+G+B}{Q+G+B}=\frac{d_2}{d_1}\cdot\frac{\frac{G+S_2}{S_2}}{\frac{G+S_1}{S_1}} \qquad (1)$$

or, since G is always very small in comparison with Q, and B may be neglected, we have

$$Q=(R+G)\frac{d_1}{d_2}\cdot\frac{S_2}{S_1}\cdot\frac{G+S_1}{G+S_2} \qquad (2)$$

The resistance Q is made up of two factors; one the resistance of the oil film itself; the other the resistance of the insulating pieces which are used to separate the two plates. It has been found by direct experiment that the resistance of these pieces is so high that the measured resistance Q may be considered as produced by the oil film alone.[1]

In order to determine whether variation in the current strength produced any variation in the resistance of the oil film, each film was tested with four currents, *i. e.*:

1st. That produced by one Lelanche element, E. M. F. about 1.4 volts.

2d. Two Lelanche elements in series, E. M. F. about 2.8 volts.

3d. Three bichromate elements in series, E. M. F. about 5.5 volts.

4th. Twelve cells of storage battery in series, E. M. F. about 25 volts.

[1] The specific resistance of sheet rubber is nearly half a million times greater than that found for the oil under test, and the resistance of paraffine is even higher.

The results are given in the tables which follow. As far as could be determined, there was no systematic difference corresponding to the differences in electromotive forces in the circuit.

When the thickness of the oil film was less than 0.″01 some trouble was experienced from a gradual change in resistance, seemingly due to polarization of the oil film. Thus with a film 0.″01 in thickness with three bichromate cells in series and a 1-10 shunt in the galvanometer circuit, successive readings at intervals of one minute gave:

deflection right—368, 367.5, 367, 366.5, 365, 364.5, 363.5, 363, etc.

deflection left (battery reversed)—135, 136.5; after considerable interval—137.

To prove that this was not due to any change in the battery current R was brought into circuit. The deflection both right and left remained perfectly steady for many minutes.

As the fall was very slow (only ½ div. or less than ½ per cent. of the whole deflection in a minute), only a very small error will be committed if the deflection is taken as soon as the oscillation of the needle due to closing the key K (which, as the needle is strongly damped, takes place at the end of the second or third swing, or in from 20 to 30 seconds), has ceased. It is hardly necessary to add that in determining d_1 and d_2 the usual precaution of reversing the battery current at each observation and taking the mean of the two successive deflections (right and left) was always adopted. At frequent intervals the galvanometer circuit was also tested by reversal to be sure that there were no errors introduced by thermal effects.

In the case ultimately under consideration (that of measuring the resistance of a film of oil between the moving surface of a shaft and the journal box), polarization would of course be prevented by a constant movement of the layer of oil.

Determination of Specific Resistance.

In order to obtain the specific resistance from the measurements of Q we must also know:

1. The area of the conducting film. This will be of course, the area of the plates, less the area of the supporting pieces, which was always 0.5 sq. in.

2. The thickness of the film. The thickness of the film was determined by the measurement of the thickness of the separating pieces. These pieces were measured very carefully by a Betts measuring machine reading to 0.″0001. This measurement, however, might not always give the true average thickness of the film of oil for the following reasons:

a. Because of the compression of the pieces when in use by the weight of the upper surface plate, a compression which is increased by the preliminary heating and subsequent contraction of the oil and plates, the contraction of the oil film drawing the plates together and compressing the separating pieces in some instances by as much as .001 of an inch, as determined by measurement before and after using. To avoid this error the measurements taken were always those which were made after the pieces had been used.

b. Because of inequalities in the thickness of different pieces. The error due to this cause is very small, as care was always taken to select pieces which were originally of as nearly the same thickness as possible. The differences found on the measuring machine never amounted to more than two per cent., and were generally much less. Hence the average of the pieces could be taken as representing the average thickness of the oil film.

c. Because of the deflection of the plates between the points of support. This deflection, although exceedingly small, cannot be considered as negligible when compared to the thickness of the very thinnest of these films, *i. e.*, about 0.″004. To prevent this effect as far as possible, the supporting pieces are so placed that their distance from the center lines of the plates was $l/2\sqrt{3}$, where l is the distance across the plate at right angles to the centre lines.

d. Because of inequalities in the surfaces themselves. Though the two surfaces are in general accurately plane, nevertheless when considered in detail they consist of a number of points which can readily be seen and distinguished by rubbing the plates together for a short time, until the points of contact become bright and polished. Between these points are shallow cavities due to the scraping. The error due to this cause is of contrary sign to those *a* and *c*, and its effect will be particularly considered later.

If A denotes the effective area of the oil film, a its thickness, and

Q_t the measured resistance at the temperature t, we have for the specific resistance (C. G. S. units) σ_t at this temperature, if A and a are expressed in inches,

$$\sigma_t = 2.54\ Q_t \frac{A}{a} \tag{3}$$

For the specific resistance at 0° or at any standard temperature t_0, we have

$$\sigma_0 = 2.54\ Q_t \frac{A}{a} f(t-t_0) \tag{4}$$

where $f(t-t_0)$ is the temperature coefficient of the oil.

To determine this function the oil in the tank was heated to an initial temperature of about 115° C., and then, after the whole had begun to cool uniformly a series of simultaneous readings of temperature and resistance were taken while the oil and plates were cooling. As before, two thermometers were always used; one immersed in the oil tank and the other placed in a cavity filled with water in the upper surface plate. The oil was kept thoroughly stirred, and the mean of the two readings which (except for the higher temperatures while the plates were cooling rapidly) differed by only a slight amount, taken as representing the temperature of the oil film. For convenience these measurements were made on the thinnest oil film used (thickness about 0.″005), but there is no reason to suppose that the results would have been different for any of the other films.

Dimensions of Apparatus.

The size of the surface plates as measured by a Brown & Sharpe Vernier caliper was 4.45 by 5.95 inches. The total area of the plates was therefore 26.5 sq. in. The insulating pieces were always cut of such size that their combined area was as already stated 0.5 sq. in. The area of the oil film was therefore 26 sq. in. The films tested were of four different thicknesses, as follows:

Series A.—The insulating material was sheet rubber. Three pieces were used, of the following thicknesses:

No. 1. Average of six measurements gave . . .	0.″01822
No. 2. " " " " " . . .	0.″01790
No. 3. " " " " " . . .	0.″01827
Average thickness of three pieces (=average thickness of oil film)	0.″01813

Series B.—The insulating material was sheet rubber. Four pieces were used of the following thicknesses:

No. 1. Mean of six settings gave	0.″0601
No. 2. " " " " "	0.″0613
No. 3. " " " " "	0.″0602
No. 4. " " " " "	0.″0615
Average thickness of pieces (=average thickness of film)	0.″0608

Series C.—The insulating material was paraffined paper. Four pieces were used of the following thicknesses:

No. 1. Mean of six settings gave	0.″01019
No. 2. " " " " "	0.″01017
No. 3. " " " " "	0.″01030
No. 4. " " " " "	0.″01017
Average thickness of pieces (=average thickness of oil film)	0.″01021

Series D.—The insulating material was paraffined paper. Four pieces were used, but unfortunately they were accidentally destroyed before they were measured on the Betts measuring machine. They had been measured by micrometer caliper before being used, and the thickness so obtained, which was 0.″0075, is probably correct within two or three per cent.

Series E.—Combined observations for specific resistance and temperature coefficient. The material used in this case was paraffined paper. Four pieces were used of the following thicknesses:

No. 1. Mean of six settings gave	0.″0046
No. 2. " " " " "	0.″00463
No. 3. " " " " "	0.″00467
No. 4. " " " " "	0.″00461
Average thickness of pieces (=average thickness of oil film)	0.″00462

The results of the measurements with these different thicknesses and with the varying strength of battery current are given in the following tables:

TABLE I. (SERIES A.)

Battery.	Remarks.	Shunt. $\frac{S_1 + G}{S_1}$	Shunt. $\frac{S_2 + G}{S_2}$	Mean d_1	Mean d_2	Q_t from (2).	t*	Q_0 from (7).	δ
						megohms		megohms	
1 Leclanche cell. . . .	. . . . .	$\frac{1}{1000}$	1	28.75	61.5	11.97	16° C.	10.53	−.10
" " . . .	Plates separated and pieces rearranged.	$\frac{1}{1000}$	1	29.25	62.75	11.93	16	10.49	−.14
" " . . .		$\frac{1}{1000}$	1	29.40	64.50	11.67	17	10.58	−.05
" " . . .	Pieces rearranged again.	$\frac{1}{1000}$	1	29.50	65.25	11.59	17	10.51	−.12
" " . . .		$\frac{1}{1000}$	1	29.70	66.25	11.48	17.5	10.57	−.06
" " . . .	. . . . .	$\frac{1}{1000}$	1	29.50	66.25	11.40	18.0	10.66	+.03
2 " " (series)	. . . . .	$\frac{1}{1000}$	$\frac{1}{10}$	91.00	20.65	11.28	"	10.55	−.08
" " "	. . . . .	$\frac{1}{1000}$	$\frac{1}{10}$	91.75	21.00	11.19	"	10.47	−.16
3 Bich. " "	. . . . .	$\frac{1}{10000}$	$\frac{1}{10}$	28.85	67.35	10.97	20	10.97	+.34
" " "	. . . . .	$\frac{1}{10000}$	$\frac{1}{10}$	28.75	67.40	10.92	20.5	11.11	+.48
Storage (12 cells). . .	. . . . .	$\frac{1}{10000}$	$\frac{1}{100}$	98.75	22.50	11.23	18	10.52	−.11
" " . .	Pieces rearranged again.	$\frac{1}{10000}$	$\frac{1}{100}$	98.50	22.50	11.21	"	10.51	−.12
" " . .		$\frac{1}{10000}$	$\frac{1}{100}$	98.45	22.10	11.41	18+	10.68	+.05
1 Leclanche cell. . . .	Taken 24 hrs. later after completely rearranging apparatus.	$\frac{1}{1000}$	1	36.00	83.35	11.06	19	10.69	+.06
" " . . .		$\frac{1}{1000}$	1	36.50	84.90	11.01	"	10.65	+.02

Mean resistance Q_0 at 20° C. 10.63+
Probable error of mean (15 observations) ±.030

* Temperatures were not observed as closely as they should have been this first day.

TABLE II. (SERIES B.)

Battery.	Remarks.	Shunt. $\frac{S_1+G}{S_1}$	Shunt. $\frac{S_2+G}{S_2}$	Mean d_1	Mean d_2	Q_t from (2).	t	Q_0 from (7)	δ
						megohms		megohms	
Storage battery. . . .	. . . . .	$\frac{1}{1000}$	$\frac{1}{10}$	97.75	75.75	33.03	18° +C.	30.90	—
" " . . .	. . . . .	$\frac{1}{1000}$	$\frac{1}{100}$	98.35	7.65	32.92	18.5	31.32	—
" " . . .	After standing some time undisturbed.	$\frac{1}{1000}$	$\frac{1}{10}$	96.30	69.50	35.48	18.5	33.76	+.125
" " . . .	Pressure on top plate.	$\frac{1}{1000}$	$\frac{1}{10}$	97.60	71.75	35.00	18.7	33.51	−.125
" " . . .	Pressure removed.	$\frac{1}{1000}$	$\frac{1}{10}$	96.25	69.85	35.28	"	33.77	+.135
" " . . .	Pieces rearranged.	$\frac{1}{1000}$	$\frac{1}{10}$	96.85	70.25	35.29	"	33.77	+.135
3 Bich. cells (series). .	. . . . .	$\frac{1}{1000}$	$\frac{1}{10}$	28.25	20.75	34.84	18.8	33.47	−.165
" " " .	. . . . .	$\frac{1}{1000}$	$\frac{1}{10}$	28.45	20.85	34.92	"	33.55	−.085
2 Leclanche cells (series)	. . . . .	$\frac{1}{100}$	1	97.35	71.75	34.74	19	33.59	−.045
" " "	. . . . .	$\frac{1}{100}$	1	97.50	72.40	34.48	"	33.35	−.285
1 Leclanche cell. . . .	. . . . .	$\frac{1}{100}$	1	36.80	27.45	34.33	19.5	33.75	+.115
" " . . .	Places separated and	$\frac{1}{100}$	1	36.90	27.50	34.36	"	33.77	+.135
" " . . .	pieces rearranged.	$\frac{1}{100}$	1	36.90	27.55	34.28	"	33.70	+.065

Mean resistance Q_0 at 20° C. 33.635
Probable error of mean (11 observations) ±.030

TABLE III. (SERIES C.)

Battery.	Remarks.	Shunt. $\frac{S_1+G}{S_1}$	Shunt. $\frac{S_2+G}{S_2}$	Mean d_1	Mean d_2	Q_t from (2).	t	Q_0 from (7).	δ
						megohms		megohms	
3 Bich. cells (series)	Only three pieces used . .	$\frac{1}{1000}$	$\frac{1}{10}$	28.5	111.05	6.57	22° C.	7.06	+.20
" " "	. . . . .	$\frac{1}{1000}$	$\frac{1}{10}$	28.35	111.75	6.50	22	6.98	+.12
" " "	Pieces readjusted end for end.	$\frac{1}{1000}$	$\frac{1}{10}$	28.45	115.50	6.31	22	6.78	−.08
2 Leclanche cells "	. . . . .	$\frac{1}{100}$	$\frac{1}{10}$	99.70	40.50	6.31	22	6.78	−.08
1 " cell. . . .	. . . . .	$\frac{1}{100}$	1	39.75	161.50	6.31	22.5	6.90	+.04
" " . . .	Fourth piece placed under plate.	$\frac{1}{100}$	1	39.75	166.00	6.12	23.0	6.82	−.04
" " . . .	Pressure applied to top plate.	$\frac{1}{100}$	$\frac{1}{10}$	39.75	16.85	6.04	23.0	6.73	−.13
" " . . .	. . . . .	$\frac{1}{100}$	$\frac{1}{10}$	39.75	17.45	5.84	23.5	6.64	−.22
" " . . .	After standing 12 hrs.	$\frac{1}{100}$	$\frac{1}{10}$	43.15	18.00	6.14	22.0	6.60	−.26
" " . . .	Pieces rearranged.	$\frac{1}{100}$	$\frac{1}{10}$	43.40	18.00	6.17	22.5	6.75	−.11
" " . . .	. . . . .	$\frac{1}{100}$	$\frac{1}{10}$	43.40	18.05	6.17	22.5	6.75	−.11
" " . . .	After standing 6 hrs.	$\frac{1}{100}$	$\frac{1}{10}$	44.50	14.80	7.70	17.8	7.16	+.30
" " . . .	Pieces rearranged.	$\frac{1}{100}$	$\frac{1}{10}$	44.50	14.85	7.68	17.8	7.14	+.28
" " . . .	. . . . .	$\frac{1}{100}$	1	44.00	150.00	7.51	17.9	7.00	+.14
" " . . .	Heated to temp. about 40° and cooled.	$\frac{1}{100}$	$\frac{1}{10}$	43.70	22.25	5.02	25.0(?)	—(?)	——

Mean resistance Q_0 at 20° C. 6.86+
Probable error of mean (14 observations) ±.032

TABLE IV. (SERIES D.)

Battery.	Remarks.	Shunt. $\frac{S_1+G}{S_1}$	Shunt. $\frac{S_2+G}{S_2}$	Mean d_1	Mean d_2	Q_t from (2)	t	Q_0 from (7)	δ
						megohms		megohms	
3 Bich. cells (series) .	Only 3 pieces used.	$\frac{1}{1000}$	$\frac{1}{10}$	27.80	154.00	4.62	20° C.	4.62	−.36
" " " .	Plates separated and pieces readjusted.	$\frac{1}{1000}$	$\frac{1}{10}$	27.75	148.50	4.78	19.9	4.76	−.22
" " " .	Pieces readjusted and fourth piece added.	$\frac{1}{1000}$	$\frac{1}{100}$	27.75	146.00	4.86	19.8	4.82	−.16
" " " .	. . . . .	$\frac{1}{1000}$	$\frac{1}{10}$	27.75	143.50	4.95	"	4.91	−.07
" " " .	After standing some time.	$\frac{1}{1000}$	$\frac{1}{10}$	27.60	138.50	5.10	19.5	5.01	+.03
" " " .	Pieces readjusted.	$\frac{1}{1000}$	$\frac{1}{10}$	27.50	140.75	5.00	"	4.92	−.06
" " " .	After standing 12 hours undisturbed.	$\frac{1}{1000}$	$\frac{1}{10}$	28.05	117.50	6.00	15.8	5.20	+.22
" " " .	Pieces readjusted.	$\frac{1}{1000}$	$\frac{1}{10}$	28.15	121.50	5.93	16.2	5.21	+.23
2 Léclanche " .	. . . . .	$\frac{1}{100}$	$\frac{1}{10}$	103.50	48.75	5.44	17.0	4.93	−.05
" " .	Pieces readjusted.	$\frac{1}{100}$	$\frac{1}{10}$	106.00	49.25	5.51	17.1	5.00	+.02
1 Léclanche cell . . .	. . . . .	$\frac{1}{100}$	$\frac{1}{10}$	45.65	20.90	5.59	"	5.08	+.10
" " . . .	. . . . .	$\frac{1}{100}$	$\frac{1}{10}$	45.65	21.00	5.57	17.4	5.10	+.12
" " . . .	One piece taken out and others readjusted.	$\frac{1}{100}$	$\frac{1}{10}$	45.75	21.25	5.51	"	5.05	+.07
" " . . .	Pieces readjusted again.	$\frac{1}{100}$	$\frac{1}{10}$	45.85	21.75	5.40	17.6	4.99	+.01
" " . . .	Pressure applied.	$\frac{1}{100}$	$\frac{1}{10}$	46.10	21.75	5.48	"	5.06	+.08
" " . . .	" removed.	$\frac{1}{100}$	$\frac{1}{10}$	45.50	21.50	5.42	17.8	5.04	+.06

Mean resistance Q_0 at 20° C 4.98

Probable error of mean (16 observations) ±.026

TABLE V.[1] (SERIES E.)

Shunt. $\frac{S_1+G}{S_1}$	Shunt. $\frac{S_2+G}{G}$	Mean d_1	Mean d_2	Q_t from (2)	t mean.	Q_0 from (7)
				megohms		megohms
$\frac{1}{1000}$	$\frac{1}{1000}$	45.85	35.85	0.327	88°5 C.	
"	$\frac{1}{1000}$	45.35	29.25	0.397	77.5	
"	$\frac{1}{1000}$	45.75	25.10	0.467	73.25	
"	$\frac{1}{1000}$	45.75	23.60	0.496	71.0	
"	$\frac{1}{1000}$	45.75	21.25	0.551	67.0	
"	$\frac{1}{1000}$	45.75	17.60	0.666	61.25	
"	$\frac{1}{1000}$	45.70	16.45	0.711	59	
"	$\frac{1}{1000}$	45.70	13.75	0.851	55	
"	$\frac{1}{1000}$	45.70	12.95	0.903	53.25	
"	$\frac{1}{1000}$	45.70	9.55	1.225	42.5	
"	$\frac{1}{100}$	45.70	93.50	1.251	42.0	
"	$\frac{1}{100}$	45.80	85.50	1.371	40.9	
"	$\frac{1}{100}$	45.80	71.25	1.646	35.9	
"	$\frac{1}{100}$	45.80	68.50	1.712	34.85	
"	$\frac{1}{100}$	45.85	65.00	1.806	34.60	
"	$\frac{1}{100}$	45.85	62.25	1.886	33.40	
"	$\frac{1}{100}$	45.85	53.00	2.215	29.20	
"	$\frac{1}{100}$	45.90	49.75	2.362	27.80	
"	$\frac{1}{100}$	45.95	46.50	2.530	26.45	
"	$\frac{1}{100}$	45.95	44.25	2.66	25.00	3.21
"	$\frac{1}{100}$	45.95	42.25	2.76	24.00	3.21
"	$\frac{1}{100}$	45.95	41.75	2.82	23.50	3.20
"	$\frac{1}{100}$	45.10	33.05	3.49	16.8	3.14
"	$\frac{1}{100}$	45.10	33.00	3.50	16.8	3.15

Mean resistance Q_0 at 20° C. 3.18+

In order to reduce the observed resistance at different temperatures to a standard temperature it is necessary to first determine the temperature coefficient from the observations of Series E. For this purpose these observations were first plotted (Fig. 2) and by the aid of a flexible steel strip a smooth curve was drawn, which represented most closely the experimental results. To represent this curve empirically an equation of the usual series form, *i. e.*,

$$Q_t = Q_0 \left[1 + A_1(t-t_0) + A_2(t-t_0)^2 + A_3(t-t_0)^3\right] \qquad (5)$$

[1] One Leclanche cell used throughout. Last two observations made 48 hours later, after plates had been separated and put together again.

was chosen, three terms being judged sufficient to represent the results over the range of temperature observed (90° to 15° C). In order to determine the values of Q_0, A_1, A_2, and A_3, the theoretically best method is to form a series of observation equations by substituting in (5) the observed values of Q_t and t, and deduce from them four normal equations by the method of Least Squares. But in

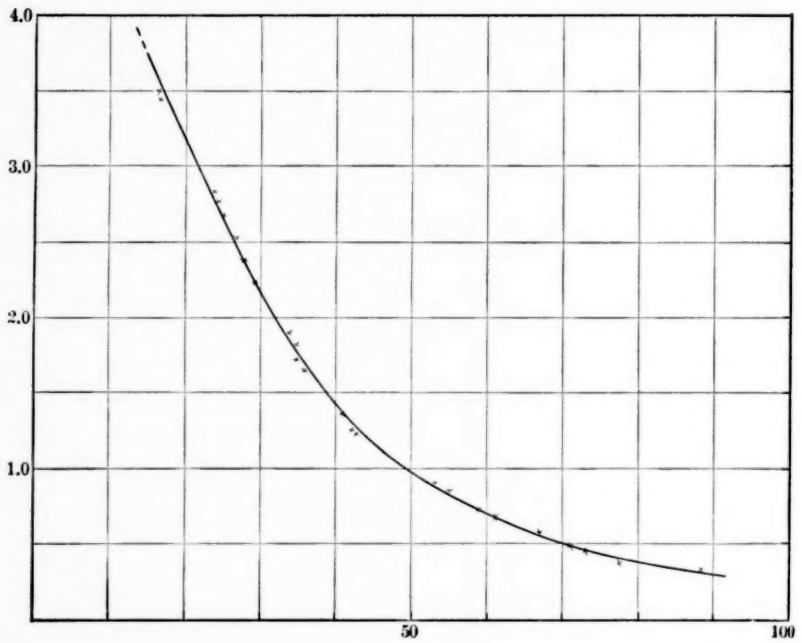


Fig. 2.

this case the observed points fell so nearly on the smooth trial curve that it was judged sufficient to determine the values of these constants directly from four carefully selected points on the trial curve. The points so selected were those corresponding to the four temperatures of 20°, 30°, 50°, and 80°. The temperature selected as standard for these experiments was 20°, as being about the mean temperature of all the series taken, and as therefore involving the least chance of error in the reductions of the observations to this temperature. The value of Q_0 is therefore at once determined. From inspection of the trial curve it was found to be

$$Q_{20^\circ}=3.18 \text{ megohms.}$$

This value agrees almost exactly with that afterward found in reducing Series E.

For other points we find

$$Q_{30^\circ}=3.18\,(1+10\,A_1+100\,A_2+1000\,A_3)=2.15$$
$$Q_{50^\circ}=3.18\,(1+30\,A_1+900\,A_2+27000\,A_3)=.983$$
$$Q_{80^\circ}=3.18\,(1+60\,A_1+3600\,A_2+216000\,A_3)=.370$$

The values of A_1, A_2 and A_3, determined from these equations, are respectively

$$A_1=-0.038$$
$$A_2=+0.00061$$
$$A_3=-0.0000037$$

Equation (5) thus becomes

$$Q_t=3.18\left\{1-0.038\,(t-20^\circ)+0.00061\,(t-20^\circ)^2-0.0000037\,(t-20)^3\right\} \qquad (6)$$

The curve represented by this equation between $t=15^\circ$ and $t=100^\circ$ is plotted in Fig. 2. By inspection it will be seen that it represents the results of experiment very well; probably almost if not quite as well as it would had the values A_1, A_2, and Q_0, all been determined by the theoretically more precise, but far more laborious process of Least Squares. Indeed, in this case the amount of labor involved (as in many others where the Least Square process is commonly employed), would have been vastly disproportionate to the results obtained, and was, therefore, unwarranted by the accuracy of the observations themselves.

Between the points $t=15^\circ$ and $t=25^\circ$ (between which all the observations of the first four series lie) the curve corresponds almost exactly to a straight line, and can therefore be represented by a simpler expression of the form.

$$Q_t=Q_0\,(1-\beta t) \qquad (7)$$

The mean value of β between these limits, as determined from the tangent to the curve at the point $t=20^\circ$, is

$$\beta = 0.0343.$$

The temperature coefficient of the oil is represented by the expression in the parenthesis of (6), or

$$f(t-20^\circ) = 1 - 0.038\,(t-20^\circ) + 0.00061\,(t-20^\circ)^2 - 0.0000037\,(t-20^\circ)^3 \qquad (8)$$

or between 15° and 25° by the simpler expression from (7)

$$f(t-20^\circ) = 1 - 0.0343\,(t-20^\circ). \qquad (9)$$

By the aid of (9) the observations in the five tables have been reduced to the chosen standard temperature of 20°, and the means, taken as representing the mean resistance of the five films at 20° C. As before stated, three and in some cases four sets of measurements were made in each series, with different strengths of battery current. It will also be noted from the remarks that the plates were separated and the insulating pieces readjusted several times during each set of measurements, that the conditions of experiment were varied in many other ways (such as applying and then removing pressure from the top plate, heating and cooling the plates, when in position, varying the number of supporting pieces between the plates, etc.), and that each series extended over two or three days. Under these circumstances, and considering also the number of observations, it may be fairly considered, I think, that in the mean the accidental errors of experiment have been eliminated. For the same reasons the probable errors deduced from each set may be taken as fairly representing the accuracy of the results attained. None of the observations have been omitted save the first two in Series B, which were obviously considerably in error (probably sufficient time was not given the plates to cool down after the first setting, as it will be noticed that the resistance rose to a value very near the mean "after standing for some time undisturbed;") and one in Series C, where the temperature was uncertain, having been taken too soon after heating. There are several that might have been rejected on Chauvenet's criterion and the apparent accuracy thereby considerably increased, but such as arbitrary treatment of the results of physical measurement on purely mathematical grounds, and where there has been no apparent cause of suspicion registered at the time of making

these particular observations, has always seemed to me a very dangerous and unwarranted proceeding.[1]

An inspection of these probable errors (see Table VI.) shows, as might have been expected, that the accuracy of the determination decreases with the thickness of the film of oil. The error in the case of the thick film (Series B) is less than one-tenth of one per cent., while the largest error in the case of the two thin films is only half of one per cent., a degree of accuracy considerably greater than is necessary for the purpose for which these observations were originally made.

TABLE VI.

Series.	Mean resistance 20°.	Error in per cent.	Thickness.	Specific resistance at 20°; megohms per cu. cm.
B	33.635 ± 0.030	0.0009	0.0608	36500
A	10.63 ± 0.030	0.0025	0.01813	38700
C	6.86 ± 0.032	0.0046	0.01021	44300
D	4.98 ± 0.026	0.0051	0.0075	43800
E	3.18	about 0.005 [2]	0.00462	45400

When we calculate the specific resistance from these results we find an interesting result; viz., that the specific resistance increases regularly as the thickness of the film decreases. This is shown in Table VI., and better in Figs. 3 and 4. In Fig. 3 the means of the different series are plotted as a function of the thickness, a, and in the second the absolute specific resistances calculated from these first results are plotted in the same way. With the exception of the one result from Series C, the points all lie on a smooth curve, which in the first case passes through the origin, and in the second appears to be asymptotic to the two axis of σ and a. The variation is of too regular a character to be accidental. I had indeed anticipated that such a result might be obtained before beginning the experimental work, and it was for that reason that I chose to work on films as thin as I could conveniently obtain rather than on the oil in mass.

[1] I have since had no reason to change this opinion so far as reduction of one's own observations are concerned. When it is a question of combining a large number of observations by different observers, as is necessary in Coast Survey work, the case is somewhat different.

[2] As judged from the divergence of the observations from a smooth curve.

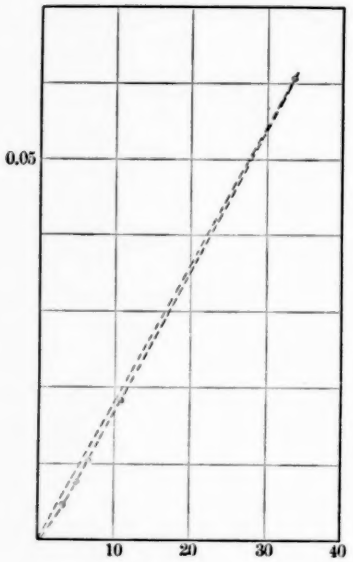


Fig. 3.

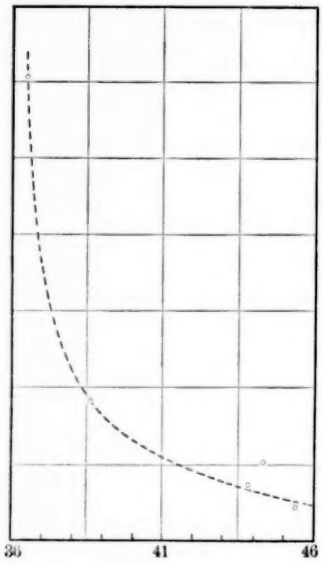


Fig. 4.

A change in specific resistance is what we might expect with decrease of thickness, because of the effect of one surface upon the other, particularly when the thickness of the film approaches the range of molecular attractions. No such limit has yet been reached here; yet it would seem that such an effect is beginning to be felt. It is possible that these results may be explained by considering that the effective thickness of the oil film is greater than the thickness of the supporting pieces on account of the cavities due to the scraping of the plates, although such an effect is directly opposed to the effect of causes (a) and (c), (see p. 80) both of which tend to *decrease* the thickness of the film. In order to determine whether this explanation is sufficient to account for the differences I have observed, I have assumed that the effective thickness of each film is $a+x$ where x is the mean correction required to the measured thickness a, on account of the cavities in the plates. To determine x we take the two extreme values of Q_{20} given in Table VI., and equate the values of σ obtained from them. This gives us

$$\underset{\text{Series B.}}{\frac{33.635}{0.0608+x}} = \underset{\text{Series E.}}{\frac{3.18}{0.00462+x}}$$

Whence $x=0.00125$

For the corrected thickness and specific resistance we then obtain

Series.	Corrected thickness.	Corrected sp. resist. Megohms per cu. cm.	δ
B	0.″06205	35750	0
A	0.01938	36200	450
C	0.01146	39400	3650
D	0.00875	37500	1750
E	0.00582	35750	0

From this table it will be seen that while the first and last results are reconciled by this assumption, all the others are still large, and what is more, the divergence is systematic and symmetrical. If the differences were fully accounted for by the assumed correction x to the thickness, the positive and negative differences from the mean ought to have been irregularly distributed.[1]

The most conclusive way, however, of determinating whether this explanation is the true one or not is to use two surfaces which have been ground together after scraping, so as to obtain two perfectly *smooth* as well as *true* surfaces. Unfortunately, I was not able to do this, because the plates were the property of the mechanical department. As soon as possible I intend to make a pair of plates for myself, and repeat the experiments.[2] In conclusion I wish to ex-

[1] A further objection to this method of explaining the observed variation in σ_0 is the large value of x necessary. According to Reynolds (see paper "On the Theory of Lubrication, and its Application to Mr. Tower's Experiments," Phil. Trans., Vol. 177, § 18, p. 184), the irregularities in a well scraped surface ought not to be greater than 0.″0001, and the maximum correction, therefore (supposing the hollows in the two plates to all come opposite each other! ! !), ought not to be more than 0.″0002 at the most; only 1/6 that which it is found necessary to assume.

[2] When I spoke of these results to Professor Michelson about a year after they had been obtained, he told me that he had found a similar result in the case of gold or silver films, (I have forgotton which), *i. e.*, that the specific resistance of the metal as determined from these thin films was very much higher than the values obtained from wires or strips. This strengthened my belief still further in the reality of the observed change. Still further evidence in this direction has been afforded by the recent results of Reinhold and Rücker on the electrical resistance of soap films. In their first work on this subject these authors found (see Phil. Trans., 1883, Section A.) that in soap films made from a solution containing 3 per cent. of KNO_3 in solution, the specific resistance "is the same whether the liquid be examined in bulk or in the form of a film 12$\mu\mu$ (0.″0000005) in thickness." But more recently (Proc. Royal Soc., Vol. 53, p. 304) they have found that when the salt is omitted from the solution, the specific resistance changes with change in thickness, although in this case it diminishes, *i. e.*, the electrical conductivity increases, as the film grows thinner.

press my thanks to Professor Thomas for the use of his instruments and laboratory, and to Professor Robinson for the loan of the surface plates.

PHYSICAL LABORATORY, OHIO STATE UNIVERSITY,
COLUMBUS, December, 1888.

Notes on the Determination of the Thickness of the Film of Oil in the Bearings of Revolving Shafts.

In 1888, while working in the mechanical laboratory of the Ohio State University, the question came up as to the actual thickness of the film of oil in the bearings of a shaft rotating at different speeds.[1] At that time I was unable to find any experimental data on the subject, and as the question seemed one of considerable practical as well as theoretical interest, I spent some time in devising plans by which the problem might be experimentally attacked under actual working conditions. Of the methods which suggested themselves at the time, that which seemed the most promising and practical was to measure the electrical resistance between the box and the shaft when the latter was in motion and at rest. In the latter case the shaft is practically in metallic contact with its box, and the resistance is very low; but in the latter it is, according to Reynolds' results, always separated from it by a thin film of oil, whose thickness and therefore whose resistance will depend on the speed of the shaft and the pressure on the box. In order to determine from the measured resistance the thickness of the film it is necessary to know the specific resistance of the oil, and the temperature coefficient (to determine which the experiments described in the preceding paper were undertaken), and also the area over which the film is spread and its temperature.

It is further necessary to know whether the thickness of the film is uniform, or if not, in what way it varies.

As regards the temperature of the oil, this is easily determined by means of a thermometer placed in a deep cavity in the metal of the box or shaft as near the bearing surface as possible, the cavity being filled with water, or preferably with mercury. The mass of the metal surrounding the bearing surfaces is so great in comparison

[1] See paper by Professor Osborne Reynolds, Theory of Lubrication, etc., already referred to, (foot note on p. 93).

with the mass of the oil, and its conductivity is so high that the temperature recorded by the thermometer could be but very little if any lower than the temperature of the oil film itself, which is the seat of the heating.[1] The area of the film is, of course, the total area of the bearing between box and shaft. The point most difficult to determine is as to whether the film is of equal thickness or not, over the whole of the bearing surface. Two distinct causes, which may be classed respectively as irregular and regular, tend to introduce differences in thickness between different parts of the oil film. The first includes such irregularities as are due to roughness or inequalities in the surfaces of the box or shaft, or to the introduction of particles of metal by abrasion, etc. The first cause of irregularity may be avoided by choosing for the experiments a shaft and journal box which have first been carefully fitted to each other, and then allowed to run together, first in one direction then in the other, a sufficient length of time under light pressure to produce perfectly smooth and polished surfaces; the second by carefully cleaning the box and shaft before beginning the experiments, with benzine, and then during the measurements keeping the bearing well supplied with clean, fresh oil from a suitable reservoir.

The regular causes of variation in thickness are, first, the differences in pressure in different parts of the journal box. The effect due to differences in pressure can be very nearly eliminated for the purposes of experiment by applying a pressure to the top box which is equal to the pressure exerted on the box by the weight of the shaft and attached mechanism, or by using, as was done in this case, in the experiments which were begun, a Thurston oil testing machine,[2] in which the journal brasses are brought together by a spring which acts simultaneously and equally on both top and bottom brasses. A further advantage of using this machine is that the work lost in friction and therefore the heat developed in the box in any given length of time may be accurately measured in each experiment. The second cause of variation, *i. e.*, that due to eccentricity of the

[1] This last quantity may also be indirectly determined from measurements of the frictional resistance, f, in the manner indicated by Reynolds, loc. cit.

[2] For description see Professor Thurston's "Friction and Lost Work in Machinery and Mill Work," John Wiley & Sons, New York; also Pratt & Whitney's Catalogue. 1893 ed., pp. 289–91.

shaft in the box, is produced by the variation in the thickness of the film itself. If the bearings are perfectly fitted to each other when they are in actual contact (see Fig. 5), then when the two are separated slightly the opening between the two is greater at the bottom *a* than at the top *b*. In practice, however, the two are more likely to be concentric when slightly separated than when actually together, because that is the condition under which wear between the two takes place. Hence if, as is nearly always the case, the box is softer than the shaft and the two are allowed to run together any length of time at a given pressure equally applied to both boxes, the surfaces will at that pressure be nearly if not quite concentric, particularly if the sides of the box are slightly relieved as shown in dotted lines in Fig. 5, and the direction of motion is frequently reversed.[1]

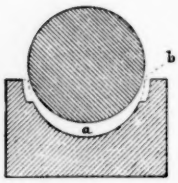


Fig. 5.

As has already been stated, these experiments were interrupted before the apparatus for the resistance measurements had been set up and put in working order. Some preliminary results, however, had been secured by placing an ordinary (comparatively insensitive) tangent galvanometer directly in series with a resistance box, and the revolving shaft, a single gravity cell being used to furnish the current.

Qualitative results were secured, showing that the resistance was measurably increased even when the pressure was very great and the speed very slow.[2] The deflection of the galvanometer decreased very rapidly (showing rapid increase of resistance) when the speed was raised or the pressure diminished. Under any one condition the deflection was as constant as could have been expected, considering the conditions of the experiment, there being at the time no provisions for a constant supply of oil or for the maintenance of constant speed.

[1] Further consideration respecting this problem will be found in another paper by the writer "On the Theory of Lubrication and the Determination of the Thickness of the Film of Oil in Journal Bearings," which will be published in the October or November number of the Journal of the Franklin Institute.

[2] This confirms Reynolds conclusion that a film of oil is always present between the moving surfaces.

Addendum.

While writing these notes another plan of measurement has occurred to me which would be useful as a check on results obtained by the electrical method, although it has not the advantage of the latter in allowing results to be obtained under the same conditions as usually obtain in machine bearings. This would be to have a long conical bearing, Fig. 6, accurately fitted to a solid box B, whose longitudinal position with respect to the revolving shaft A can be accurately located by means of a microscope C mounted on the box, and focussed on a fine circumferential line on the shaft. The thickness of the oil film in this case is determined by the longitudinal motion of B on A. Differing pressures between shaft and box may be produced by a spring or weighted lever acting on the end of the box at P.

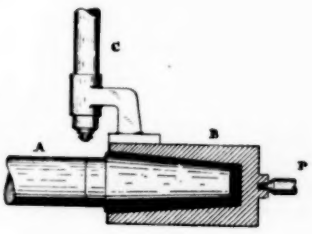


Fig. 6.

Yerkes Observatory, March, 1897.

CERTAIN OPTICAL AND ELECTRO-MAGNETIC PROPERTIES OF HEAT WAVES OF GREAT WAVE-LENGTH, I.

BY H. RUBENS AND E. F. NICHOLS.

IN a recent paper[1] the present writers described a method by which it is possible to obtain fairly homogeneous heat rays of great wave length without the aid of the usual spectrum apparatus. The method was based upon the principle that if the total radiation from a source emitting waves comprising a great range of different wave-lengths be reflected from a surface, the reflected rays will, in general, have a composition different from the incident rays. Waves which correspond to regions of heavy absorption in the substance of the reflecting body, and which it in consequence reflects as a metal, will have greatly the advantage over all other rays in the reflection spectrum, and in the remainder, after several successive reflections on surfaces of the same material, these rays only will be found in measurable quantity.

The matter contained in the present paper may, for convenience of treatment, be divided into four parts. The first part gives in detail the changes in the arrangement of apparatus by which a great improvement in both the sensitiveness and the accuracy of measurement was obtained. The second part contains the account of results obtained by the application of the improved method to a wider range of substances. The third part deals with the absorption reflection and refraction of long heat waves in a variety of media. The fourth part describes a number of experiments which reveal the electro-magnetic character of heat waves of great wave-length.

I.

Improvements in Apparatus.

The changes introduced in the order of apparatus described in the earlier paper were: First, the substitution of a Linnemann zircon-

[1] PHYSICAL REVIEW, IV., p. 513, 1897, and Naturw. Rundsch., 11 Jahr'g., No. 43, 1896.

ium burner instead of the incandescent platinum foil coated with powder. By this change long waves in somewhat greater intensity were obtained, doubtless, because the zircon could be heated to a much higher temperature than the powders of such substances as fluorite for example. Second, in the interest of greater purity and homogeneity of the rays obtained, four or even five reflecting surfaces were used instead of three. Third, in place of the bolometer of the earlier study a torsion radiometer was substituted. The last two of these substitutions led to the much more complicated arrangement of apparatus shown in Fig 1.

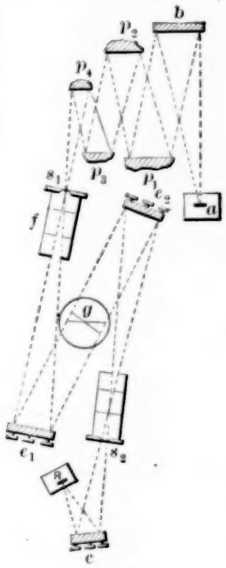


Fig. 1.

a is the zircon plate of the Linnemann burner, *b* a concave silver mirror of great focal length, $p_1\ p_2\ p_3\ p_4$ plane surfaces of the substance studied. The reflecting spectrometer $s_1\ c_1\ c_2\ s_2$, upon the table *g* of which a coarse wire grating could be placed for wave-length measurements, was so arranged that either a bolometer or radiometer could be used and Fig. 1 shows the arrangement of apparatus chosen for the radiometer. In this arrangement the slit s_2, in place of the eye piece or bolometer wire, remained fixed and the arm of the spectrometer carrying the mirror c_2 could be turned through any given angle and its position accurately read from the graduated spectrometer circle. An image of the slit s_2 was projected upon one wing of the radiometer at *R* by the aid of a concave silver mirror *c* of short focal length.

In this arrangement of the spectrometer the relation between the angle α, through which the mirror c_2 moves as read from the spectrometer circle, and the diffraction angle φ is not a simple one, yet a proportionality between the two for small values of α is easily proved. The course of the middle ray in the spectrometer $s_1\ c_1\ c_2\ s_2$ is shown in Fig. 2 for the zero position, and for a neighboring position of the moveable arm c_2. Let the angle between the two positions of the mirror arm be α and the corresponding defraction angle φ. Further

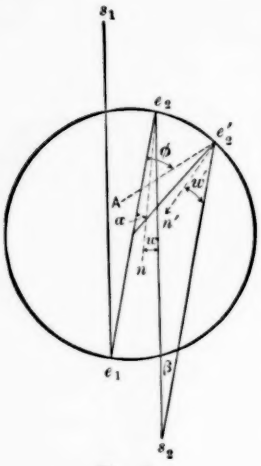


Fig. 2.

let w' represent the angle between n' the optic axis of the mirror c_2' and the ray $s_2 c_2'$, and w represent the corresponding angle between n the optic axis of the mirror c_1 and the ray $s_2 c_2$ then

$$w' = w + \alpha - \beta$$

where β represents the angle between $s_2 c_2'$ and $s_2 c_2$. The reflected ray $s_2 c_2$ corresponds to the incident ray $c_1 c_2$, the reflected ray c_2' s_2 corresponds to $A c_2'$; whence the angle φ is the difference in direction between $c_1 c_2$ and $A c_2'$, and consequently

$$\varphi = \alpha + w' - w = 2\alpha - \beta$$

From this relation it appears that the assumed proportionality between φ and α holds at best only as a close approximation for small values of α. In the present case the greatest and least values of α were comprised between $+3°$ and $-3°$. The best evidence of the trustworthiness of this assumption in the present application, lies in the observation that the diffraction images occur at equal angles on either side of the central image about which they occupy symmetrical positions. The effect of a lack of proportionality between α and β would be such that for negative values of α the differential coefficient $\partial\beta/\partial\alpha$ would be greater, and for positive values less than for $\alpha=0$. As it was impossible from the dimensions of the apparatus to establish the relation of α to β with sufficient accuracy by direct linear measurements, the spectrometer was empirically calibrated by a series of measurements in the diffraction spectra to be given later in detail.

For quite another reason the present arrangement was applicable only to small deviations. The angle of diffraction $\varphi = 2\alpha - \beta$ which in the present case equalled approximately 1.7α was such that for large values of α a noticeable displacement of the diffracted bundle of rays between the movable mirror c_2 and the slit s_2 occurred. As here used the maximum displacement amounted to ± 7 mm. which could not have caused any displacement of the observed energy maxima because the concave mirrors c_1 and c_2 were considerably

larger than the diffraction grating so that in both extreme positions the whole diffracted bundle was caught by the mirror e_2.

This arrangement of the spectrometer is without doubt less rigorous than that proposed by Wadsworth[1] which likewise permits both slits to remain fixed, yet the simplicity of the arrangement here used is in its favor.

Apparatus.

The spectrometer has been described in an earlier paper by one of the present writers[2] and the grating was the same one used in the earlier study. Suffice it here then to say that the circle of the spectrometer could be read to seconds, and the grating constant, as optically determined and as directly measured on the dividing engine, was 0.3716+mm. The grating from its peculiar construction gave only spectra of the odd orders.

By the aid of an eye-piece mounted directly behind the fixed slit s_2, with the grating on the spectrometer table at g, the empirical calibration of the spectrometer was made by reading the settings for the images of the sodium line on both sides of the central image. The results are shown in Table I.

TABLE I.

Order of Spectrum.	Deviation observed		$\frac{r+l}{2}$ observed.	Computed.	Difference δ''
	left l	right r			
5	16′ 17″	16′ 23″	16′ 20″	16′ 20″	0
9	29 45	29 45	29 45	29 42	−3
13	43 0	42 45	42 57	42 54	−3
17	56 20	56 0	56 10	56 6	−4
21	69 25	69 10	69 17	69 18	+1
25	82 35	82 20	82 28	82 30	+2
29	95 55	95 30	95 42	95 42	0
33	109 5	108 35	108 50	108 54	+4

The numbers in the table show that beyond the image of the 13th order a systematic asymmetry appears between the position of the images on the right and the corresponding images on the left, which

[1] Wadsworth, Phil. Mag., 49, p. 337, 1894.
[2] H. Rubens, Wied. Ann. 54, p. 270, 1894.

in the case of the image of the 33d order reaches 30″ of angle, but the mean values of the images right and left still agree perfectly with the computed values. The computed values correspond to the diffraction of a grating with a constant of 0.6139 mm. The ratio of this apparent constant to the real constant 0.3716 is 1.6522— a factor with which the observed angles of deviation α are to be multiplied to obtain the true diffraction angle φ. The magnitude of this factor depends on the position of the fixed slit s_2, it was, therefore, necessary to determine the position of this slit from time to time. The uncertainties produced by the slight changes in position thus observed affected the values obtained by quantities less than 0.2 per cent.

Instead of the grating described above, a similar grating of higher dispersion, made of gold wires, was often introduced, the constant of which was 0.0662+ mm.; but the inferior quality of this grating, when compared with the former one, rendered the results obtained by its aid even less trustworthy.

The torsion radiometer resembled closely the instrument previously described by one of the present writers.[1] In the new radiometer no fluorite window was used, but instead a plate of chloride of silver 2½ mm. thick in place of the mica plate. The chloride of silver was found to transmit rays of 24 μ wave-length partially.[2] The new radiometer vanes were very much smaller than in the old suspension—measuring only 6 mm. long by 1.5 mm. wide and 5 mm. across from the outer edge of the one vane to the outer edge of the other. The vanes were coated with finely powdered black oxide of iron. The disturbance of a very troublesome tremor of the radiometer suspension encountered at first, due to a tremor of the building, was completely excluded by suspending the instrument as a whole from a wall-bracket according to the method of W. H. Julius.[3] It was further possible to so completely screen the radiometer from outside thermic disturbances that the single throws,

[1] E. F. Nichols, PHYSICAL REVIEW, IV., p. 297, 1897; also Berl. Ber. 5, Nov., 1896 and Wied. Ann., 60, p. 401, 1897.

[2] The diathermancy of chloride of silver for waves of great wave-length was first discovered by C. Schulze-Sellack, Pogg. Ann., 139, p. 192, 1870. The plates used here were selected from among those made and used by Schulze-Sellack.

[3] W. H. Julius, Wied. Ann., 56, p. 151, 1895.

measured on a scale 3,000 scale divisions from the instrument, were trustworthy to 0.2 scale division. In spite of the heavy absorption of the chloride of silver radiometer window for the long waves the radiometer proved to be approximately three times as sensitive as the platinum bolometer, earlier described. If the differences shown by the two instruments in freedom from extraneous disturbances be taken into account the effective sensitiveness of the radiometer was more than five times that of the bolometer. In spite, however, of this great gain in sensitiveness of the radiometer over the bolometer, the radiometer could only be used after some material for a window had been discovered by the aid of the bolometer which transmitted partially, at least, the radiations in question.

II.

The Application of the Method.

Preliminary studies of the substances fluorite and quartz have been communicated in an earlier paper.[1] In quartz another region of metallic reflection has been found which was overlooked in the earlier study, owing to the presence of a rocksalt lens (which completely absorbed the rays of the second maximum) used to concentrate the rays from the lamp on the reflecting surfaces. The study of the short spectrum resulting from multiple reflection on fluorite surfaces has been more fully carried out with the improved apparatus which has led to more accurate results.

Substances Studied.

The method has so far been extended to the study of eleven substances including quartz, mica, fluorite, rocksalt, sylvine, crown glass, flint-glass, sulphur, potassium alum, shellac, and both rays in Iceland spar. In the first three substances regions of metallic reflection were found and the distribution of energy in the reflection maxima determined. Rays were obtained after five reflections from rocksalt, which were all but totally absorbed by the interposition of a very thin plate of the same substance. Owing to the very low intensity of these rays we have not yet succeeded in finding the position of the energy maximum in the diffraction spectrum.

[1] Rubens & Nichols, l. c.

Quartz.

The first region of metallic absorption in quartz is not so far removed from the energy maximum in the spectrum of the zirconium lamp as to make measurements of the distribution of energy in the first spectrum of rays after four-fold reflection on quartz surfaces, difficult. With the radiometer and slit widths varying from 1 to 1.5 mm. it was even possible to trace the energy curve of these rays in the higher dispension of the spectrum of the third order.

The results of such a series of observations are shown graphically in Fig. 3, in which abscissas represent angle readings of the movable mirror arm and ordinates the corresponding throws of the radiometer. In this series the slits s_1 and s_2 were 1.5 mm. wide. The greatest throw obtained in the energy maximum of the central image was 325 mm.

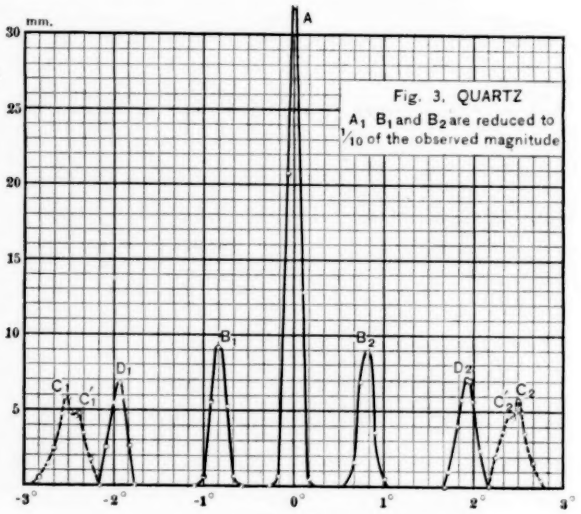


Fig. 3.

Fig. 3 shows, in all, nine maxima. Of these A is the central image, B_1, B_2 the images of the first order of the absorption band previously observed; the corresponding images in the spectrum of the third order are seen in C_1 C_1' and C_2 C_2' as dotted curves on the extreme ends. Of these the outermost C_1 and C_2 are the more intense.

In addition, the figure shows two sharply defined maxima D_1 and D_2. When a plate of rocksalt 2.6 cms. thick was interposed, the energy corresponding to these bands vanished completely while the ordinates of B_1 and B_2, C_1 C_1' and C_2 C_2' were reduced by about 10 per cent. This explains the reason why D_1 and D_2 did not appear in the results of the earlier study where, as already mentioned, a rocksalt lens was used for concentration. The distances of the maxima on either side of the central image and the wave-lengths corresponding to the settings of the spectrometer are given for two complete series of measurements in Table II. The second series does not elsewhere appear in detail.

TABLE II.

Maximum.	Angle of deviation α.		α Mean.	Wave-length λ
	Series I.	Series II.		
B_1	49′	50′	49′ 30″	8.85 μ
B_2	50	49		
C_1	2° 31½′	2° 32′	2° 31 38	9.02
C_2	2 31	2 32		
C_1'	2 24½	2 23	2 23	8.50
C_2'	2 22½	2 22		
D_1	1 56½	1 57	1 56 30	20.75
D_2	1 56½	1 56		

In accordance with these results quartz is seen to possess narrow regions of metallic reflection in the infra-red spectrum the maxima of which correspond to $\lambda = 8.50\mu$, 9.02μ and 20.75μ respectively.

The first two bands which lie close together were earlier discovered by one of the present writers.[1] The values of λ obtained at that time were 8.42μ and 8.80μ respectively of which the one is less by 1 per cent., and the other by 2.7 per cent. than the corresponding values just given. This difference is doubtless due to the increasing impurity with increasing wave-length in this region of the spectrum of the fluorite prisms in which the earlier study was made. The reason for this exists in the heavy and increasing absorption of fluorite which begins in the region under discussion. The effect of

[1] E. F. Nichols, l. c.

this sort of impurity would be to displace both maxima toward shorter wave-lengths, and for the same reason the second would suffer greater displacement than the first. This view is further confirmed by the fact that the second maximum C_1 C_2 in the present study is seen to be higher than the first, while the reverse was found in the earlier measurements. The possibility of such an error was foreseen, and attention called to it in the earlier paper.[1]

The Ketteler-Helmholtz dispersion formula, when applied to early measurements of dispersion in quartz, placed the middle of the infra-red absorption band, assumed single, at $\lambda_2 = 10\mu$.[2] In this computation the form

$$n^2 = b^2 + \frac{M_1}{\lambda^2 - \lambda_1^2} - \frac{M_2}{\lambda_2^2 - \lambda^2} \qquad (1)$$

was used.[3] It is quite possible that the new absorption band at $\lambda = 20.75\mu$ is not wholly without influence upon the dispersion in the early infra-red, or even in the visible spectrum. At any rate it is impossible to make equation (1) fit the early observations even approximately by substituting any value for λ_2 between 8.5 and 9μ where observation has placed it. On the other hand an attempt to introduce a third term of the same form as the previous two on the right side of equation (1) was more successful. With this addition the formula stands

$$n^2 = b^2 + \frac{M_1}{\lambda^2 - \lambda_1^2} - \frac{M_2}{\lambda_2^2 - \lambda^2} - \frac{M_3}{\lambda_3^2 - \lambda^2} \qquad (2)$$

In which $\lambda_2 = 8.85\mu$— the wave-length of the approximate center of absorption as shown by the mean value of B_1 and B_2 in Table II. λ_3 was put equal to 20.75μ— the wave-length of the middle of the absorption band D_1. Equation (2), while it contains the same number of arbitrary constants earlier used,[4] is in harmony with the new measurements as well as with the old.

Table III contains the observed values of the refractive index for

[1] E. F. Nichols, l. c. PHYS. REV., p. 308.

[2] H. Rubens, Wied. Ann. 54, p. 476, 1895.

[3] Wherein n is the index of refraction, λ the wave-length and b^2, M_1, M_2, λ_1^2, and λ_2^2 are arbitrary constants.

[4] In this form the arbitrary constants are M_1, M_2, M_3, b^2 and λ_1.

a number of wave-lengths in the ultra-violet, visible, and infra-red spectrum which are compared with corresponding values computed by equation (2), in which the constants M_1 and λ_1 are those computed by Ketteler,[1] and b^2 was obtained by the aid of the equation,

$$b^2 = a^2 + \frac{M_2}{\lambda_2^2} + \frac{M_3}{\lambda_3^2}$$

and the value a^2 given by Ketteler.[2] The two remaining constants M_2 and M_3 were computed from the observed indices for wave-lengths 2.84μ and 4.20μ.

TABLE III.[3]

Wave-length λ	n obs.	n comp.	Diff. $\delta \cdot 10^4$	Wave-length λ	n obs.	n comp.	Diff. $\delta \cdot 10^4$
0.19881	1.65070	1.65077	+0.7	1.969	1.5216	1.5216	0
0.27467	1.58750	1.58757	+0.7	2.84	1.5039	1.5039	0
0.35818	1.56400	1.56395	−0.5	3.63	1.4799	1.4801	+2
0.4341	1.55387	1.5539	0	4.20	1.4569	1.4569	0
0.5350	1.54663	1.5466	0	5.0	1.417	1.412	− 50
0.6563	1.54181	1.5419	+1	5.8	1.368	1.339	−290
0.7682	1.53893	1.5390	+1	6.45	1.274	1.241	−330
1.160	1.5329	1.5329	0	7.0	1.167	1.096	−710

$b^2 = 4.57877$

$M_1 = 0.010654$ $M_2 = 44.224$ $M_3 = 713.55$

$\lambda_1^2 = 0.010627$ $\lambda_2^2 = 78.22$ $\lambda_3^2 = 430.56$

The agreement between values of n computed from (2) and those obtained indirectly—from observation of reflection—is not close, yet the divergence would be fully explained by the assumption of the presence of from 1 to 2 per cent. of the energy belonging to $\lambda = 1.4\mu$ in this part of the spectrum, or of 0.3 per cent. of stray rays from $\lambda = 8.5\mu$. The former wave-length corresponds approximately to the energy maximum in the spectrum of the zirconium burner, the second to the middle of the neighboring absorption band in quartz.

[1] E. Ketteler, Wied. Ann. 30, p. 312, 1887.

[2] Compare H. Rubens, Wied. Ann. 53, p. 268, 1894.

[3] The dispersion measurements between wave-lengths $\lambda = 0.19881$ and 0.35818μ are taken from Sarasin, between $\lambda = 0.4341$ and 4.20μ from Rubens and from $\lambda = 5.\mu$ to $7.\mu$ from Nichols.

A second and more probable reason is doubtless to be found in the particular form of equation (2) which makes it inapplicable in the immediate neighborhood of a band of metallic absorption. As early as $\lambda = 8\ \mu$ the formula gives negative values for n^2 so that the limitations of the formula would appear to be the more reasonable explanation of the disagreement.

Mica.

Doubtless on account of the large percentage of silica[1] which enters into the composition of mica, rays after four reflections or surfaces of this substance yield a spectrum which resembles strikingly that of quartz just described. Fig. 4 contains the results of a series of observations of rays sent out from the zirconium lamp which, after four reflections on plane surfaces of mica, were analysed by the grating and radiometer. In this series the slits s_1 s_2 were opened 2½ mm. wide.

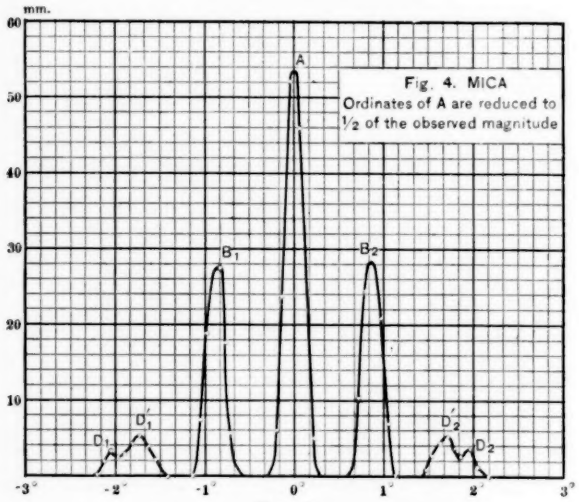


Fig. 5.

In addition to the central image A the curve shows three maxima on either side, where regions of metallic reflection exist. That the images all belong to the spectrum of the first order, and that D_1 D_1' and D_2 D_2' are not repetitions of B_1 and B_2 in the spectrum of the

[1] According to the observations of Rammelsberg mica contains from 45 to 47 per cent. of silica (Neumann, Mineralogie).

third order, is that the former vanished when a rocksalt plate 2.6 cm. thick was introduced into the path of the rays. Table IV gives the exact positions and the corresponding wave-lengths for the several maxima.

TABLE IV.

Maximum.	Angle of deviation α	α Mean.	Wave-length. λ
B_1	51′	51½′	9.20 μ
B_2	52′		
D_1	2° 0′	1°59′	21.25
D_2	1 58		
D_1'	1 43	1 43′	18.40
D_2'	1 43		

With the exception of the maximum at $\lambda = 18.40\mu$, the absorption bands correspond very closely to those observed in quartz. Because of the wide slits, and the low dispersion of the grating it was impossible to draw any conclusion concerning the distribution of energy in the maxima B_1 and B_2.[1] In consequence, these two bands were compared with the corresponding quartz bands by the aid of the grating of gold wires already mentioned which gave a higher dispersion. The results thus obtained are shown in Fig. 5.

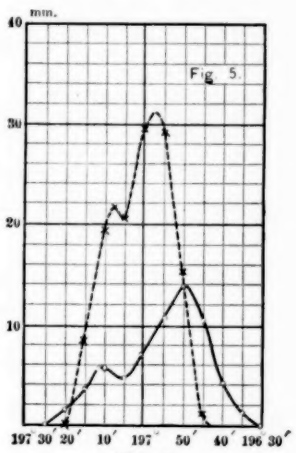


Fig. 5.

The lower curve gives the distribution of energy in the mica band, and the dotted line the same for quartz under the same circumstances. The setting for the central image in both cases was 199° 55′ 0″. From Fig. 5 it appears that the reflective power of quartz in this region is much greater than that of mica. It is further seen that the elevations B_1 and B_2 consist of two separate maxima, the corre-

[1] The images in the spectrum of the third order could not be studied because too weak in energy.

sponding wave-lengths of which do not agree with the similar maxima in quartz. The wave-lengths of the maxima in mica are 8.32 μ and 9.38 μ, and those for quartz 8.52 μ and 9.03 μ respectively. The last two values are in close agreement with the results of the quartz series given in Table II.

The natural conclusion from these results is, that the metallic reflection of mica, with perhaps the exception of the maximum at 18.40 μ, is due to the silica which it contains. The small differences in the position of the reflection maxima are doubtless wholly due to an effect, already well known through optical studies—the influence of the solvent upon the absorption spectrum of the substance dissolved.

Fluorite.

The apparatus and method used in the earlier determination of the metallic reflection of fluorite have been fully described elsewhere.[1] In the present repetition of that study the following essential changes have been introduced, which require attention: First, the source of energy has been changed from an incandescent sheet of platinum coated with powdered fluorite to a zirconium lamp. Second, the number of reflecting surfaces was increased from three to four. Third, the arrangement of the spectrometer was changed. Fourth, the radiometer with vanes coated with black oxide of iron was substituted for the bolometer coated with platinum black. Fifth, a plate of vitreous chloride of silver, in the form of a radiometer window, was introduced into the path of the rays.

The results obtained with the present arrangement are shown in Table VI, while those obtained earlier appear in Table V.

TABLE V.

Series.	Deviation from central image.		Wave-length λ
	Right.	Left.	
I	3° 45′	3° 41′	24.2 μ
II	3 48	3 47	24.7
III	3 44	3 46	24.4
			Mean 24.4 μ

[1] Rubens and Nichols, l. c.

TABLE VI.

Series.	Deviation from central image.		Wave-length λ
	Right.	Left.	
I	2° 13′	2° 13′	23.8
II	2 12	2 13	23.7
III	2 12	2 13	23.7
			Mean 23.73

A difference appears between the earlier values and the later ones which lies altogether beyond the range of error in either case. The ground of this discrepancy was found to be a strongly selective absorption of chloride of silver. This absorption changes markedly even within the narrow limits of the short spectrum lying between 22 and 28μ. At three different points within this range, the transmission of a second silver chloride plate was studied and found to be

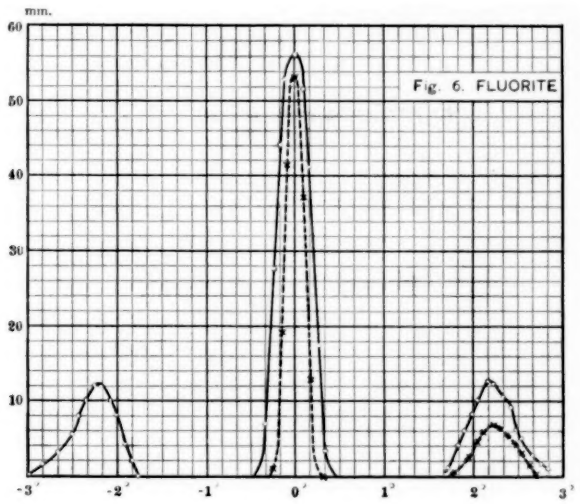


Fig. 6.

40 per cent. at $\alpha = 2° 10'$, 37 per cent. at $\alpha = 2° 15'$, and 33 per cent. at $\alpha = 2° 20'$. The combination of this transmission curve with the curve representing the true distribution of energy in the reflection maximum would be such as to displace the latter toward shorter

wave-lengths by approximately 0.°3′30″ of angle which corresponds to a wave-length difference of 0.62μ. This, when added to the observed maximum 23.73μ, gives 24.35μ as the true position of the maximum which agrees very well with 24.4μ, the value reached in the earlier study.

The first series of observations in Table VI. is shown graphically in Fig. 6. The slit widths were 3.5 mm. The radiometer throw for the middle of the central image was 56 mm., and in the maximum of the image of the first order 12 mm. was obtained. The distribution of energy is better shown by another series of observations made with slits 2 mm. wide, the results of which appear in the dotted curve in Fig. 6, which, from its form, suggests the possibility that the observed maximum consists, in reality, of two maxima belonging to a double band as in the case of quartz and mica. An attempt to effect the further separation of the two maxima was unsuccessful on account of the small quantity of energy present in these rays.

(*To be concluded.*)

MINOR CONTRIBUTION.

On the Relation Between the Osmotic Pressure and the Vapor Pressure of Solutions.

By A. A. Noyes and C. G. Abbot.

1. Introduction.

It is well known that the direct measurement of osmotic pressure is attended with great difficulty and consequently the few experiments[1] which have been carried out for this purpose have led to hardly any accurate and reliable results. Recourse has therefore been had to thermodynamic relations which make possible the calculation of this highly important quantity from more accessible data. In this way the diminution of vapor pressure, the lowering of freezing point, and other properties have been made use of. But as Nernst[2] has recently pointed out, all these thermodynamic relations give directly the osmotic work, and not the osmotic pressure itself. Since the so calculated osmotic work is the product of the osmotic pressure by a volume change which can not be experimentally measured, namely, the volume change which occurs when a definite amount of solvent is added to an infinite amount of the solution, the determinations heretofore made of osmotic work have not yielded reliable values for the osmotic pressure. The osmotic pressure has, to be sure, been calculated from the osmotic work by various investigators, but invariably with the help of certain assumptions. Thus, for example, Arrhenius[3] and Dieterici[4] have sought to determine the volume factor by investigating the volume change that occurs when to a *finite* amount of solution a certain definite amount of solvent is added. The fact that the contraction is dependent in a high degree on the quantity of added solvent should have furnished a warning against this mistake. By other authors[5] it has been assumed quite as arbitrarily that the volume change of the solution is approximately identical with the volume of the added solvent. Nernst[6] and Abegg[7] appear, on the other hand,

[1] *Pfeffer.* Osmotische Untersuchungen. *Adie,* J. Chem. Soc., *1891,* 344.
[2] Theoretische Chemie, 309.
[3] Zeitschr. für Phys. Chem., *10,* 92.
[4] Wied. Ann.
[5] Schilling, Program der K. K. Staatsoberrealschule in Olmutz für 1893–4.
[6] Theoretische Chemie, 209.
[7] Zeitschrift für Phys. Chem., *15,* 250.

to be aware of the arbitrary nature of these assumptions, and the latter entirely refrains completely from calculating the pressure from the work.

It is the purpose of this article to show from the principles of energy how osmotic pressure itself can be calculated from the diminution of vapor pressure, and to deduce the relation between it and osmotic work.

2. *Derivation of the Relation Between Osmotic Pressure and Vapor Pressure.*

Previous deductions of the relation between osmotic work and vapor pressure have all been based on the consideration of the energy changes involved in an isothermal reversible cyclical process or a virtual change of the equilibrium. For the purpose of deriving a relation involving the osmotic pressure, we will make use of the second law of thermodynamics in another form, namely, that which asserts that in a condition of equilibrium there can be no uncompensated intensity differences.[1]

Consider the system sketched in the accompanying figure and the case of equilibrium there represented. The vessel V contains a pure solvent which also fills the tube T up to the point H. At this point is placed a semipermeable wall, upon which rests a very thin layer of a solution in the same solvent. When equilibrium is reached, which comes about by a change in the concentration of the solution caused by the entrance or expulsion of solvent through the semipermeable wall, the osmotic pressure P of the solution (which in the mathematical discussions is commonly defined as the total difference in pressure between the solvent and solution[2] must evidently be equal and opposite to the hydrostatic pressure. The latter is equal to the weight of the column of solvent in the tube T diminished by the vapor pressure p_0 of the solvent and increased by that of the solution p. That is to say, if s be the specific gravity of the solvent and h_1 the height of the osmotic column above the surface of the liquid in the vessel V:

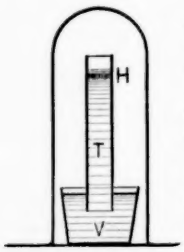


Fig. 1.

$$P = -(p_0 - p) + \int_0^{h_1} s\, dh.$$

[1] See Ostwald, Zeitschr. für Phys. Chem., 10, pp. 366, 379.

[2] This customary definition appears to us to be an unfortunate one, for, according to it, the osmotic pressure is not a single physical quantity but the algebraic sum of three independent quantics, namely, the true osmotic pressure which is exercised on the semipermeable wall by the dissolved molecules, the vapor pressure of the solvent and that of the solution which three things, for example, follow quite different temperature laws. A second disadvantage of the definition consists in the fact that the energetic relations with other quantities cannot be so simply expressed as if the rational definition of osmotic pressure was adopted. Since, however, the two quantities are different only by an amount negligible for practical purposes, we retain the customary definition to avoid confusion.

The specific gravity may be expressed with almost absolute accuracy, as follows:

$$s = s_0 - \frac{ds}{dh}(h_1 - h)$$

where $\frac{ds}{dh}$ is considered constant and s_0 is the specific gravity of the solvent at the top of the column, or practically its value under the usual pressure.

By integration, introduction of the compression coefficient $k = \frac{ds : s}{dP}$ and consideration of the fact that in the term of the second order expressing the effect of compression, it can be assumed that $s = s_0$ and $P = h_1 s$ we obtain:

$$P = h_1 s_0 (1 + \frac{P}{2} k) - (p_0 - p) \qquad (1)$$

Moreover, since the system is in equilibrium the vapors from the solvent and the solution must everywhere be under equal pressure; for otherwise a distillation would begin by which the osmotic equilibrium would be destroyed and a perpetual motion would result. Now the pressure of the vapor above the solvent falls off steadily with increasing height by an amount equal to the weight of the intervening column of vapor. That is, if δ be the specific gravity of the vapor

$$dp = -\delta dh$$

or

$$h_1 = \int_p^{p_0} \frac{dp}{\delta} = \frac{1}{M} \int_p^{p_0} v dp \qquad (2)$$

where M denotes the molecular weight and v the molecular volume of the vapor of the solvent. If, therefore, h_1 denote as before the height of the osmotic column, and p the vapor pressure of the solution, substituting for h_1 in equation (1) we obtain:

$$P = (s_0 + \frac{P}{2} k) \frac{1}{M} \int_p^{p_0} v dp - (p_0 - p) \qquad (3)$$

In this expression all the quantities of the second order are retained. In almost all practical applications, however, these are negligible, on account of their small magnitude, in comparison with the experimental error. For example in a normal solution in ether, whose compression coefficient 0.00015 is uncommonly large, the term $\frac{P}{2} k$ is only about 0.17 per cent.

An osmotic system somewhat similar to the one here described has been already employed by Duhem,[1] Gorey and Chaperon.[2] The impor-

[1] Journal de physique (2), 6, 136.

[2] Ann. de chim. et phys. (6), 13, 124.

tant difference between the two arrangements consists in that in the one case the semipermeable wall is at the top of the column and in the other it is at the bottom. The latter arrangement is unsuitable for determining the relation between osmotic and vapor pressure because the concentration of the solution varies continuously from the bottom to the top of the column as shown by Lorenz[1] and more plainly by Guglielmo,[2] thus making the osmotic and vapor pressures under consideration refer to solutions of different concentration. To neglect this consideration leads to curious conclusions, for example, that the osmotic pressure of a solution is a function of its specific gravity and that it depends on the depth of the semipermeable wall below the surface of the solvent.[3]

3. *Relation Between Osmotic Pressure and Osmotic Work.*

From equation (3) determining the osmotic pressure and expressions already obtained by different authors for the osmotic work done in pressing out a molecular weight of the solvent, the relation between these quantities follows directly. The exact expression for the osmotic work Π_m is as follows:[4]

$$\Pi_m = \int_p^{P_0} v\,dp - (p_0 V_0 - pV), \tag{4}$$

in which p, p_0 and v have their previous significance V_0 is the volume of one molecular weight of the liquid solvent, and V the volume change of the solution when this amount of solvent is added to it. By uniting this equation with equation (3) remembering that $\frac{M}{s_0} = V_0$ and that in the small correction term $(p_0 V_0 - pV)$, V_0 may be assumed equal to V, the following almost exact relation is obtained:

$$\Pi_m = P \frac{V_0}{1 + \frac{P}{2} K} \tag{5}$$

Since now in this equation the member $\frac{P}{2} K$ even with quite concentrated solutions can hardly amount to one per cent, it may be said that within the limit of error of previous experiments *osmotic pressure and work are directly proportional to each other.*

A correction on account of the contraction of the solution on mixing with the solvent such as has been applied by some authors is therefore

[1] Zeitschrift für Phys. Chem., 7, 37.

[2] Ibid, 11, 282.

[3] See Duhem (2), 6, 140.

[4] See Duhem. Jour. de Phys. (2), 6, 405; also Gorey and Chaperon, Ann. de Chim. et de Phys. (6), 13, 121.

unnecessary and incorrect. Furthermore, the conclusion of Nernst,[1] based on kinetic considerations that in the calculations of osmotic pressure from osmotic work the large volume occupied by the molecules of solvent must be taken into account, is shown by this law to be incorrect.

Finally, it may be remarked that with the establishment of this law the objection raised by Abegg[2] against the contents of an earlier article by Noyes[3] falls to the ground. The objection was based on the fact that in that article, which treated of the deviations from the gas laws in solution, the freezing point determinations were regarded as a measure of osmotic pressure instead of osmotic work. This assumption is, however, warranted so long as proportionality exists between freezing point lowering and osmotic work, and that this is the case for moderately concentrated solutions, at least within the limits of experimental error, is shown by Abegg's own calculations. The law formulated in the article of Noyes that the change of the osmotic pressure with the concentration is expressed within the limits investigated by the linear equation: $P(v-d) = K$, and that the constant d depends upon the relative size of the molecules of the solvent and of the dissolved substance retains, therefore, its validity.

BOSTON, MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
March, 1897.

[1] Theoretische Chemie, 209.
[2] Zeitschr. für Phys. Chem., 15, 254.
[3] Ibid., 5, 53.

NOTE.

Alfred Marshall Mayer.—Among American physicists devoted to pure science few have become so distinguished as the one whose death we are called upon now to record. After a period of intermittent ill health which lasted several years, but which did not cause a final interruption of duty until a few months ago, Professor Mayer died at his country home in Maplewood, New Jersey, on the 13th of July, 1897.

The family was of German descent, one of its members, Christian Mayer, emigrating to America in 1784, and becoming the head of a large and successful business house in Baltimore. Charles F. Mayer, the father of the physicist, was a gentleman of ample means and fine scholarship. Alfred Marshall Mayer was born November 13, 1836, and after receiving the usual school advantages afforded in every city, entered St. Mary's College in Baltimore. This he left at the age of sixteen years because he preferred scientific to classical training, and two years were then devoted to the study of mechanical engineering in the workshop and draughting room. Two more years were given to the laboratory study of chemistry, general and analytical, and at the age of twenty years the student was invited into the professor's chair to teach in the University of Maryland the subjects on which he had been specializing. During these years of special study in youth young Mayer formed the acquaintance of Joseph Henry, and a friendship was formed that continued until Henry's death twenty-five years afterward. It was Henry's influence and example which gave to the developing physicist a strong bias in favor of pure science, this term being used in contrast with industrial or commercial science, which is so much more popular everywhere, but particularly in America.

Professor Mayer accepted a chair in Westminster College, Missouri, in 1858, and remained there two years. Returning to Baltimore on the breaking out of the civil war, he remained for a time and then went to Paris, where he studied physics, mathematics and physiology. Most of his work was under the direction of Regnault. Returning to America in 1865 he accepted a call to the chair of physics and chemistry at Pennsylvania College, Gettysburg, and resigned in 1867 to take that of physics and astronomy at Lehigh University. In 1871 he went to Hoboken where the Stevens Institute of Technology was just organizing; and as professor of physics in this school of mechanical engineering the last twenty-six years of his life were spent.

During his student days in Baltimore Professor Mayer's first paper was

published. It was the description of a "new apparatus for the determination of carbonic acid." This was followed in 1857 by one on the estimation of minute weights by the deflection of fine glass fibres. During the next ten years he published little, but gave his time partly to routine work and partly to laying deep the foundations for the future. His fruitful scientific activity may be said to have begun with his entrance upon work at Lehigh University in 1867, and during the next ten years this activity was at its maximum. It was at first chiefly in the channels of astrophysics, and determined by the erection of an observatory which he planned and equipped. After publishing a series of observations on Jupiter he was deputed by the national government to take charge of a party of astronomers who were stationed at Burlington, Iowa, in the path of totality of the solar eclipse of August 7, 1869. A considerable number of photographs of the eclipse were secured by him, and the discussion of results was published. About the same time, however, purely physical questions were additionally engaging his attention. The first of these was on the "Thermodynamics of Waterfalls," and was occupied with the measurement of the heat generated by such cataracts as Trenton Falls and Niagara. In 1872 he devised a method of tracing the progress of a surface of conducted heat by means of the change of color with temperature observed with certain chemical substances. He applied the same method afterward to thermographs of the solar disk. In 1890 he investigated the coefficients of cubic expansion of solids determining them by observation of the temperature at which water contained in a vessel made of the solid under examination attains the same apparent volume as it has at 0°C. Among the substances thus examined was vulcanite; and a general examination of the properties of this solid followed.

At Lehigh University Professor Mayer began also some investigations on magnetism, making many observations on magnetic declination, improving on previously known methods of fixing and photographing magnetic spectra, and devising a zero-method of comparing the strengths of electromagnets. He began afterward an elaborate research on the effect of magnetization in changing the dimensions of iron and steel, and continued this for some time. He was called upon to give frequent public lectures. One of these, entitled "The Earth a Great Magnet," was delivered before the Yale Scientific Club and subsequently published. In connection with it he devised a new and very satisfactory form of galvanometer for vertical projection which attracted much notice. Another interesting demonstration which a few years afterward attracted attention was that of the configurations assumed by floating magnets when subjected to the action of a superposed magnet.

In 1874 Professor Mayer devised a new method for the study of the oscillatory nature of the spark discharge from a Leyden jar or between

the electrodes of an induction coil. Securing thus the conditions for the most nearly instantaneous discharge, he applied the induction coil with great success in the electrographic method of rating tuning forks, and in the employment of the tuning fork as a chronoscope for measuring the velocity of projectiles.

The subject of investigation in which Professor Mayer's results were most conspicuous was acoustics. His study of the tuning fork had preceded that of the spark discharge, and soon after beginning work at Hoboken he began a series of researches in acoustics which made him easily the leading authority on this subject in America. Beginning with the study of resonators, he showed how the phases of vibration in the air around a sounding body could be measured, developed a precise method of determining wave-lengths and velocities of sound in gases at various temperatures, and devised an acoustic pyrometer by which furnace temperatures were measurable with a degree of precision closely approximating that of the air thermometer. He then originated a method of determining the relative intensities of sounds under special conditions, and the absorbing power of different media for sound. The great absorptive power of India rubber led to its employment as a damper for tuning forks, and he was thus enabled to secure an approximate determination of the mechanical equivalent of an aerial sonorous vibration. He was then drawn into a study of the physiology of acoustics. He devised an ingenious method for the objective analysis of musical quality, and made some wholly new discoveries in relation to the auditory organs of the mosquito. This led to the suggestion of a modification of Helmholtz's view regarding the function of the nerve fibrils in the human ear.

Professor Mayer's most important research in acoustics was his determination of the law connecting the pitch of a sound with the duration of the residual sensation in the ear. He gave a quantitative character to what Helmholtz had reached only qualitatively. He was thus enabled to show how musical combinations that are smooth in the upper regions of the scale become rough and dissonant in the lower parts. The work was begun in 1874, repeated in 1875, and again revised with improved apparatus and the utmost circumspection in 1893. The law is expressed by an equation in which the duration of the residual sensation is given as a function of vibration frequency, and this equation is satisfied with the highest degree of precision that could be expected in any attempt to give mathematical expression to the phenomena of sensation. The importance of this law has been recognized in the leading recent books on acoustics.

Professor Mayer successfully applied acoustic methods to investigations primarily undertaken with other ends in view. During the last few years of his life he conducted an elaborate research on the variation in

the modulus of elasticity of various materials with change of temperature. From the observed pitch of a bar vibrating transversely the velocity of propagation of sound in it was found and from this the modulus of elasticity. In such a research no ordinary determination of pitch could be relied upon. He went, therefore, to Paris and secured the coöperation of Dr. Koenig, whose great tonometer is the only instrument in the world that is capable of determining pitch with severe accuracy through a sufficiently wide range. The results of the investigation showed remarkable agreement and are of great scientific value.

Other contributions to acoustics that deserve mention are Professor Mayer's discovery of what has been called acoustic repulsion and its application to the "sound mill;" an effect independently discovered afterward by Dvorâk, in Austria. He applied resonators successfully also to the location of a distant source of sound at sea, such as a foghorn. His last acoustic paper was in 1894, on the production of audible beat tones from two vibrating bodies whose frequencies are so high that the separate tones are inaudible.

In 1893 Professor Mayer became much interested in the phenomena of simultaneous color contrast, and published a research on this subject. He contributed to the general investigation of Roentgen rays in 1896. His last work, undertaken during the intervals of the disease which terminated his life, was a study of the forces acting in the flotation of discs and rings of metal and a determination, in this way, of the capillary constant of the liquid employed. The difficulties involved were great, but the results were among the best ever attained on this subject. The research was published but a few days before a stroke of paralysis put an end to all further activity.

Of Professor Mayer's charming personal qualities little can be said in this brief notice of his scientific work. Hs was a man of great native refinement and social polish. His geniality and warmth of heart made deep impressions on all who came into contact with him. The intellectual quality most conspicuous was his extraordinary ingenuity and fertility in resources. His perception was quick, and his patience in unraveling difficulties was great. He leaves a record of originality which will last, and a large circle of friends to mourn his loss.

S.

NEW BOOKS.

Elements of Mechanics, including Kinematics, Kinetics, and Statics, with Applications. By THOMAS WALLACE WRIGHT, M.A., Ph.D., New York, Van Nostrand, 1896. 8vo. 8+372 pp.

This volume is a new and considerably enlarged edition of the author's "Text-book of Mechanics," published in 1890. The directness and simplicity with which the subject is treated throughout, the restriction of the subject-matter to what is absolutely necessary for the applications to engineering, the abundant supply of well selected exercises, questions, and "applied" problems, and a certain freshness and unconventionality of style will commend it favorably to many students and teachers. While the book has grown from 260 to 372 pages, the range and scope of the work have not been essentially widened; nor has the general method of treatment been changed very much, except in so far as the author seems to have been influenced by Professor A. G. Greenhill's unpublished *Notes on Dynamics* which we are told in the preface were placed at his disposal for the preparation of this new edition. The last mentioned fact adds a special interest to Professor Wright's volume.

The book is divided into nine chapters as follows: I. Kinematics, motion (pp. 6–43); II. Matter in motion, Newton's laws of motion (pp. 44–71); III. Dynamics of a particle (pp. 72–140); IV. Statics of a body (pp. 141–205); V. Friction (pp. 206–223); VI. Work and energy (pp. 224–270); VII. Dynamics of rotation (pp. 271–319); VIII. Elastic solids (pp. 320–337); IX. Metric units (pp. 338–367).

All questions requiring the use of three dimensions of space are rigorously excluded. Thus, even in the "Statics of a body" the six general conditions of equilibrium are not given, but only the conditions for coplanar forces; the first chapter does not extend beyond the kinematics of a point; and the seventh treats only of rotation about a *fixed* axis. For an elementary treatment this restriction to one and two dimensions is perfectly proper and very convenient as it simplifies matters enormously. The question may, however, be asked whether such a narrow treatment of mechanics is sufficient for the student of engineering for whom this work appears to be mainly designed. Even if it were admitted that the engineer never meets in actual practice with problems that cannot be reduced to the two-dimensional treatment, it would seem as if the students in our best engineering colleges ought to be given a broader view

of the science of mechanics. The restriction of the theoretical part of mechanics to its narrowest limits, as we find it in such books as Professor Wright's, is due partly to the desire to avoid the use of higher mathematics, partly to the tendency to introduce numerous applications so as to make the course in mechanics really a course in *applied* mechanics. As regards the first point it may be said that the amount of pure mathematics taught in the larger engineering schools is quite sufficient for basing on it a thorough course in mechanics; indeed, it is through the course in mechanics that this mathematical knowledge is best assimilated and made so familiar as to become really useful. With respect to the second point it should be remembered that it is more important to make the student understand and appreciate the broad fundamental principles of the science than to communicate to him a large variety of particular applications. It is only just to state, however, that in Professor Wright's book the applied problems constitute a particularly valuable feature.

In the first chapter a certain lack of precision in language makes itself felt more than in the rest of the work. This laxness may be due to an effort to be "practical" and to avoid mathematical forms of expression. But a student who has mastered as much trigonometry, analytic geometry, and calculus as is necessary for reading the book intelligently, will appreciate accuracy and profit by it. He will have no difficulty in perceiving that we can speak only of the path of a point or particle, but not (without special definition) of "the path of a body." He will readily understand the application of the terms "constant" and "variable" to a velocity as soon as he knows how velocity is measured. "Uniform motion" should have been defined; and "constant velocity" would be preferable to "uniform velocity." In Art. 22, p. 18, line 5, the words "with constant velocity" should be omitted; in Art. 23, p. 19, read "arithmetically" twice instead of "algebraically;" in Art. 31, p. 26, for "polar axis" read "radius vector." A more serious confusion of ideas occurs in Art. 30, p. 25. Here d^2s/dt^2 is treated as if it represented the total acceleration of a point moving in a curve, and the rectangular components of the acceleration are derived in an erroneous way. This confusion could hardly have arisen if the general expressions for tangential and normal acceleration were derived; but these important expressions are not given. In the introduction the author states his preference for the term "Phoronomics," instead of "Kinematics," although he uses generally the latter term. What he had in mind was probably the term "Phoronomy," advocated by W. H. Besant (see *Nature*, Vol. 45, p. 462, and the preface to the second edition of Besant's Dynamics, 1893) and used by a number of other writers (Hermann, Kant, Budde, etc.).

In his discussion of Newton's laws of motion, the author takes a less

critical and independent position than he did in the first edition. He deals with these laws by the philological method of interpreting a text, or rather of interpreting various meanings into the text, not by the scientific method of free investigation. The interpretation of the first law appears particularly strained. Thus, our author says (p. 45): "The law lies beyond our experience, as we have no experience of one body not acted upon by another." Have we any experience of one body acted upon by another? Is not all such action pure hypothesis? "Our direct experience goes, however, a certain distance in confirmation of the law * * *," and then he proceeds to show that observation and experiment tend to confirm the law. But the statement that a particle does not change its state of rest or motion without some cause is merely a truism; for where there is no effect there is no need for inventing a cause; and the statement that any change in motion is brought about by force merely supplies a name for the cause, at least as long as force is not otherwise known and defined. Newton's first law is, therefore, not the expression of an experimental fact; it merely assigns the name force to the cause producing change of motion (more exactly, of momentum). On the same page the author says: "From the law we learn that rest and motion are equally states of a body, the body being wholly without influence on its rest or motion. This property of matter is called *inertia* * * *." That the body, *i. e.*, the mass of the body, is wholly without influence on its rest or motion is hardly correct; otherwise, why should we take the mass of the body into account in mechanics? What the author means is, of course, that matter does not change its motion by itself, *i. e.*, without some cause. But is not this a truism? It must, however, be admitted that it is by no means an easy task to give a fully satisfactory presentation of the fundamental laws of motion suited to an elementary book.

Professor Wright has evidently made a careful study of the question of standards and units; his work contains a large amount of valuable information on this subject. In the matter of "weight" and the units of mass and force he follows Professor Greenhill; from page 57 on, all the m's are changed into w/g. Gravitation units are used throughout, the "metric units" being reserved for the final chapter. Considering the elementary character of the work, this is perhaps the best plan. But the rapid advance of electrical engineering may perhaps be counted upon to bring the absolute units into better repute among the engineers of the future.

Finally, attention might be called to the excellent articles (96–100) on the motion of projectiles which probably also owe something to Professor Greenhill.

ALEXANDER ZIWET.

Theory of Physics. By JOSEPH S. AMES, Ph.D. 8vo, pp. xviii+513. New York, Harper & Brother, 1897.

The reading of this book was undertaken with the lively interest that one has in a new book upon one's own subject coming from a leader in one of the foremost institutions. To introduce a student into the domain of Physics is no mean task, and he who can furnish the clearest and most stimulating introduction has done much toward promoting the growth and development of the science. Here as elsewhere it can be said that "Well begun is half done."

In the reading of the text, it has been kept in mind that the book presupposes no previous training on the part of the student or at most a very elementary one. It is divided as follows: Book I., Mechanics and Properties of Matter; Book II., Sound; Book III., Heat; Book IV., Electricity and Magnetism; Book V., Light.

The chapter on kinematics treats motions of translation and rotation in a somewhat general way, makes a sharp distinction between velocity, a directed quantity, and speed, a mere number, which seems rather difficult to maintain, for the author on page 24 speaks of an acceleration, or directed quantity, as being opposed to a speed, and touches lightly upon harmonic motions and the spinning top. Harmonic motion hardly receives the share of attention its importance in sound and light seems to demand.

Dynamics is devoted to the consideration of matter in translation, rotation and equilibrium, to work, energy and mechanics, and to a short but valuable treatment of wave-motion. A marked feature of the style of treatment is the tendency to begin with a generalization and deduce from it the special cases, as the "Principle of the Conservation of Momentum" is taken for the starting point in the consideration of matter in translation, and the "Principle of the Conservation of the Moment of Momentum" for matter in rotation. From a logical point of view, this leaves nothing to be desired, but many teachers will question whether this is the easiest and surest way for an immature student to approach the subject for the first time. The successful effort made to connect the formulæ for matter in rotation with the corresponding ones for translation must, however, be helpful to the student.

The principle of stable equilibrium has received a very wide application in the book and illustrates again the author's fondness for applying general principles to particular cases, as it is applied to dissociation, evaporation, expansion, fusion and solution. Book I. is completed by a short chapter on gravitation and one on "Properties of Size and Shape of Matter" in which several formulæ are introduced "which may be deduced by the help of the higher mathematics."

Sound is defined as a sensation and the theory of sound in physics as

the study of the *nature* of this *sensation* and the exact conditions under which it is produced. After defining sound as a sensation, it must be a little confusing to the careful student to read, "if the velocity of sound in any gas is known, etc." Light also is defined as a pure sensation, yet we read "if red light had a greater velocity than blue, etc." It is certainly a question whether physicists ought to surrender these useful terms which have long stood for the physical bases of the corresponding sensations and for whose use in this sense we have the sanction of the standard dictionaries, and substitute for them the awkward circumlocutions that would be necessary. It ought to be mentioned that Döppler's principle is given correctly as it is not in several current books.

Book III. Heat furnishes a clear and logical presentation of the study of the "so-called heat-effects," and closes with a short but good elementary treatment of the kinetic gas-theory and a two-page chapter on thermodynamics which will hardly succeed in giving the student a conception of the second law of thermodynamics.

Electricity, magnetism and light are treated almost entirely from a theoretical standpoint and here the title of the book finds its strongest *raison d'être*, though the character of the whole book is true to the purpose indicated in the title, "Theory of Physics." Much is made of lines of force and the energy of the dielectric and Poynting's theory of the flow of energy perpendicularly into a conductor is more than suggested; the dissociation theory of electrolysis is given at some length and applied to the explanation of conduction in solids.

Light is treated entirely from the side of physical optics and the principle of curvature of the wave-surfaces is used throughout the treatment of reflection and refraction. There is no question of the elegance of this method nor of the insight into wave propagation furnished by a clear understanding of it, but that it lends itself to an elementary presentation of the subject of light is somewhat doubtful. The idea of curvature is not easy for a student to grasp, and the constructions of images and the tracing of the path of the light is not so easy from this point of view, and it will be noticed that the author has drawn many diagrams in which rays are really used, though nothing is said of the change from wave to ray construction. It is trusting a great deal to the student's power of constructive imagination to reduce a lens to a straight line, as is done in most of the diagrams. This is only an apparent simplifying of the diagrams.

Throughout the book statements are made very guardedly, and few are clear cut and positive—such statements as students and many teachers like to find. We read under gravitation that "there *seems* to be undoubted evidence that each portion of matter in the universe influences the motion of every other portion," and in reference to a helix we find

that "when this form of a conductor carries a current, it is *sometimes* called a solenoid" and "a solenoid with a soft iron bar inside is *sometimes* called an electromagnet." In striking contrast to these statements is the frequent use of the phrase "of course," when it is not a matter of course at all. Few errors in the text were noticed, and these are mainly typographic; on page 103, "a piece of cork is a very rigid body;" page 238, "If the temperature of the lead is not very *hot*," and the lettering of figures 3 and 232 does not agree with the text.

The omission of problems that illustrate and tend to fix the principles enunciated in the text will be felt by many teachers; however, this gives to each teacher the chance to select problems in accordance with his own ideas of the relative importance of the principles.

CHAS. E. ST. JOHN.

The Old Light and the New. By WILLIAM ACKROYD, F.I.C. 12mo, pp. 102. London, Chapman & Hall, 1896.

Roentgen Rays and Phenomena of the Anode and Cathode. By EDWARD P. THOMPSON, M.E., E.E., etc., and PROF. WILLIAM A. ANTHONY. 8vo, pp. xiv+190. New York, D. Van Nostrand Company, 1896.

The X-Ray; or, Photography of the Invisible. By WILLIAM J. MORTON, M. D., and EDWIN M. HAMMER. 8vo, pp. 196+32 half tone plates. New York, The American Technical Book Company, 1896.

Les Rayons X et la Photographie de l'Invisible. By GEORGES VITOUX. 12mo, pp. 191. Paris, Chamuel, 1896.

It is interesting to compare these four little books which were brought forth, almost simultaneously, to meet the popular demand for X-ray literature. Those who take up Mr. Ackroyd's book in the expectation of finding therein any adequate discussion of the Roentgen phenomena will be disappointed. The volume is in fact devoted chiefly to an attempt to extend, by the aid of very inadequate data, the author's theory concerning the relations of color to chemical composition to the explanation of the varying opacities of bodies to the X-rays.

The other three volumes under consideration differ widely in their treatment of the subject. Messrs. Thompson and Anthony devote themselves chiefly to the compilation of a very useful and complete summary of the rapidly growing X-ray literature up to the time that their book went to press. The chief fault of their work may be said to be the lack of arrangement; but indeed, it is difficult to imagine in what way the heterogeneous mass of observations which were published during the first months after the announcement of Röntgen's discovery could have been

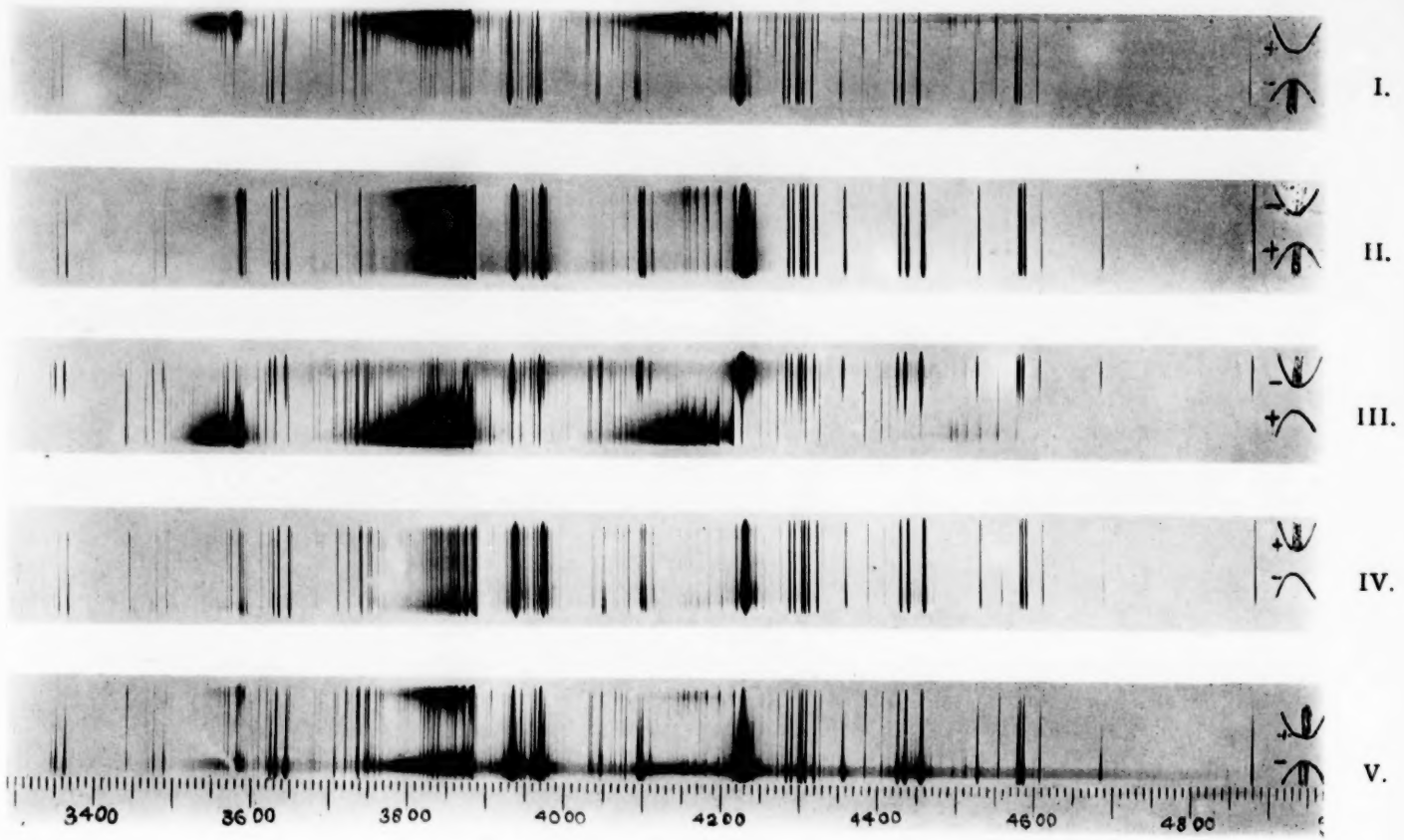
arranged. The book owes its usefulness chiefly to the fact that it gives brief and intelligible abstracts of the numerous papers which had been published and that reference is made in every case to the originals. The authors deserve the thanks of the scientific public, likewise, for refraining to reiterate in their book the familiar electrical principles upon which production of the X-ray depends.

He who opens the volume of Morton and Hammer at its first page will think that he has happened by mistake upon one of those saddening *primers of electricity* with which the book market has been flooded of recent years. Nor will he be disabused of his mistake until nearly sixty pages have been perused. If, however, he perseveres until he reaches Part III., entitled "Operation," he will have his reward. From this point onward the book takes on a useful form. The description of the arrangement of apparatus for X-ray photography is definite, and impresses one at once as the work of those who write out of their own experience. The chapter treating of the relation of the vacuum to the production of X-ray tubes containing, as it does, a detailed description of methods of both increasing and diminishing the vacuum by artificial means, is especially useful.

By far the most readable of the books under consideration is the treatise of Georges Vitoux. The author approaches the subject in a systematic manner. He assumes a reasonable elementary knowledge on the part of his readers, and confines himself to a well-arranged historical review of his subject. An adequate idea of the contents of this interesting little volume can be best given by taking it up chapter by chapter.

The author first describes the discovery of the X-rays, giving a vivid account of the popular excitement produced by Roentgen's announcement. He devotes his second chapter to the translation of Roentgen's original paper into French, and then proceeds to a discussion of the results obtained by earlier workers in this field. It may be questioned whether it was wise to include in this list the unsuccessful attempts of Reichenbach to demonstrate by means of the daguerreotype the existence of the *odylic effluvia;* but with this exception, the historical sketch is admirable. It is followed by chapters upon the nature and origin of the X-rays, their properties and their relation to fluorescence. The remainder of the volume is devoted to the technique of operation and to the applications of X-ray phenomena. It is a question whether the brief and non-commital statements concerning "*black light*" photography in the dark, and by electricity, and the so-called "*photography of thought*," might not have been omitted to advantage.

E. L. NICHOLS.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